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Improving the Teleportation of Superposition of Entangled Coherent States

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Author's contribution

This work is based on the teleportation of non-maximally entangled coherent states and using the scheme of some of my previous work which is already mentioned in the reference.

Original Research Article

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ABSTRACT

I consider the teleportation of superposition of coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ using beam splitters and phase shifters. I propose some modifications which consist of writing the output state as a superposition of the vacuum state and nonzero even photon state, which allow for an almost perfect teleportation to be achieved for an appreciable mean number of photons.

Keywords: Entangled states; fidelity; teleportation.

1. INTRODUCTION

Quantum teleportation [1-5] is the disembodied transport of a physical system which requires, as a prerequisite, the distribution of entangled quantum states between a sender and a receiver. The sender is required to make a joint measurement on her state and an unknown quantum system, thereby obtaining classical information that she transmits to the receiver. Using this information, the receiver transforms his state to recover the original unknown system. Entanglement [6] is one of the major ingrediant necessary for teleportation to take place and it is also very important in other QIP tasks like quantum teleportation [7-9], quantum computation [10], quantum dense coding [11] and quantum cryptography [12].

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In 2003, Enk and Hirota [7], proposed a scheme of teleporting 1 bit of information contained in a superposition of odd and even coherent states, using entangled coherent states, a beam splitter and two phase shifters. The authors used maximally bipartite entangled coherent states which play the role of the quantum channel, and concluded that the probability for successful teleportation is 1/2. In Ref.[5], Wang proposed a teleportation scheme of bipartite ECS with maximally tripartite ECS as the quantum channel and concluded that the probability for successful teleportation is only 1/2 and showed that the photon counts in one of their two outputs are always zero and that if the photon counts in the other output are odd, perfect teleportation is possible and concluded that the success probability is 1/2. Here, in this paper I take a non-maximally entangled coherent states and show that writing the output state as a superposition of the vacuum state and nonzero even photon state, I show that near perfect teleportation with almost perfect fidelity is possible. In this paper I consider a superposition of coherent states $|\alpha\rangle$ and $|-\alpha\rangle$, and with our suggested modification show that the fidelity of teleportation is improved to almost unity for an appreciable mean number of photons.

1.1 Teleportation Scheme

In this paper I consider two parties: Alice, and Bob. Let the state which Alice wishes to teleport to Bob be $|I\rangle_{1,2} = \in_+ |\alpha,\alpha\rangle_{1,2} + \in_- |-\alpha,-\alpha\rangle_{1,2}$ in the mode 0. I may write the initial state $|I\rangle_{1,2}$ in terms of even and odd coherent states [13] as,

$$|I\rangle_{1,2} = A_+ |EVEN,\alpha\rangle_0 + A_- |ODD,\alpha\rangle_0 = \cos\frac{\theta}{2} |EVEN,\alpha,\alpha\rangle_0 + \sin\frac{\theta}{2} e^{i\phi} |ODD,\alpha,\alpha\rangle_0 , \quad (1)$$

where

$$|EVEN,\alpha\rangle = \frac{|\alpha\rangle + |-\alpha\rangle}{\sqrt{2\left(1+x^4\right)}}, \quad |ODD,\alpha\rangle = \frac{|\alpha\rangle - |-\alpha\rangle}{\sqrt{2\left(1-x^4\right)}} . \quad (2)$$

Here, I can relate constants $A_\pm$ to $\in_\pm$ by,

$$A_\pm = (\in_+ \pm \in_-)\sqrt{(1 \pm x^4)/2}, \quad \in_\pm = [2(1+x^4)]^{-1/2} A_+ \pm [2(1-x^4)]^{-1/2} A_- . \quad (3)$$

and θ and ϕ are defined by, $\tan\frac{\theta}{2} e^{i\phi} = A_- / A_+$

I consider the teleportation of state $|I\rangle_{1,2}$ using the entangled coherent state of the type,

$$|E\rangle_{4,5,6} = \left[2\left(1-x^8\right)\right]^{-1/2}\left(\left|\sqrt{2}\alpha\right\rangle_4 |\alpha\rangle_5 |\alpha\rangle_6 + \left|-\sqrt{2}\alpha\right\rangle_4 |-\alpha\rangle_5 |-\alpha\rangle_6\right) , \quad (4)$$

in the modes 3, 4 and 5. The initial state of the composite system is then given by $|\Psi\rangle_{1,2,3,4,5} = |I\rangle_{1,2} |E\rangle_{3,4,5}$. Modes 1, 2, and 3 remains with Alice while modes 4 and 5 goes directly to Bob. Mode 2 passes through phase shifter PS-I which changes the state $|\alpha\rangle_2$ to the state $|-i\alpha\rangle_6$. Alice then mixes states in modes 1 and 6 by using a 50:50 lossless beam splitter which changes [14] an input state to state $|\beta\rangle_0|\gamma\rangle_3$ to state $\left|\frac{\beta+i\gamma}{\sqrt{2}}\right\rangle_4\left|\frac{\gamma+i\beta}{\sqrt{2}}\right\rangle_5$. Now, the state 7, 8, 3, 4, 5 Ψ becomes the direct product of the vacuum state $|0\rangle_8$ and a state of modes 7, 3, 4, 5. Now the state 8 is separated from the remaining system. Then Alice allows mode 7 to pass through phase shifter PS-II which changes the state $|\eta\rangle_7$ to the state $|-i\eta\rangle_9$. She again mixes modes 9 and 3 using a beam splitter which changes an input state to state $|\chi\rangle_9|\delta\rangle_3$ to state $\left|\frac{\chi+i\delta}{\sqrt{2}}\right\rangle_{10}\left|\frac{\delta+i\chi}{\sqrt{2}}\right\rangle_{10}$. Finally, she allows one of the output, say, mode 10 to pass through phase shifter PS-III which changes a state $|\lambda\rangle_{10}$ to the state $|-i\lambda\rangle_{12}$. The complete process changes state $|\Psi\rangle_{9,3,4,5}$ to state,

$$|\Psi\rangle_{11,12,4,5} = \left[2\,(1-x^8)\right]^{-1/2}\Big[\in_+\Big(|2\alpha\rangle_{11}|0\rangle_{12}|\alpha\rangle_4|\alpha\rangle_5 + |0\rangle_{11}|-2\alpha\rangle_{12}|-\alpha\rangle_4|-\alpha\rangle_5\Big) + \in_-\Big(|0\rangle_{11}|2\alpha\rangle_{12}|\alpha\rangle_4|\alpha\rangle_5 + |-2\alpha\rangle_{11}|0\rangle_{12}|-\alpha\rangle_4|-\alpha\rangle_5\Big)\Big]. \quad (5)$$

Naresh Chandra et al. [15-18] modified the photon counting scheme by expanding the state $|\pm\sqrt{2}\,\alpha\rangle$ in terms of vacuum, non-zero even (NZE) and odd number states as

$$|\pm\alpha\rangle = \sqrt{x}\,|0\rangle + 2^{-1/2}\,(1-x\;)\,|NZE,\,\alpha\rangle \pm [(1/2)(1-x^2)]^{1/2}\,|ODD,\;\alpha\rangle, \quad (6)$$

where

$$|NZE,\alpha\rangle = \left(|\alpha\rangle + |-\alpha\rangle - 2\sqrt{x}|0\rangle\right)/\sqrt{2}\,(1-x) \quad (7)$$

is the part of states with non zero even number of photons only.

Enk and Hirota and Wang used the expansion,

$$|\pm\sqrt{2}\,\alpha\rangle = \sqrt{\frac{1}{2}\left(1+x^4\right)}\,|EVEN,\sqrt{2}\,\alpha\rangle \pm \sqrt{\frac{1}{2}\left(1-x^4\right)}\,|ODD,\sqrt{2}\,\alpha\rangle, \quad (8)$$

which makes difference in the classification of result of photon-counting and in the planning for unitary transformations to be performed by Bob.

For more precise calculations, I chose equation (6) over (8), and the final output state becomes

$$
\begin{aligned}
|\Psi\rangle_{11,12,4,5} = \; & \frac{x^2}{\sqrt{2\left(1-x^8\right)}} |0\rangle_{11} |0\rangle_{12} (\in_+ + \in_-) \left[|\alpha,\alpha\rangle_{4,5} + |-\alpha,-\alpha\rangle_{4,5} \right] \\
& + \frac{1}{2}\sqrt{\frac{1-x^4}{1+x^4}} \left\{ |NZE, 2\alpha\rangle_{11} |0\rangle_{12} \left[\in_+ |\alpha,\alpha\rangle_{4,5} + \in_- |-\alpha,-\alpha\rangle_{4,5} \right] \right. \\
& \left. + |0\rangle_{11} |NZE, 2\alpha\rangle_{12} \left[\in_+ |-\alpha,-\alpha\rangle_{4,5} + \in_- |-\alpha,-\alpha\rangle_{4,5} \right] \right\} \\
& + \frac{1}{2} \left\{ |ODD, 2\alpha\rangle_{11} |0\rangle_{12} \left[\in_+ |\alpha,\alpha\rangle_{4,5} - \in_- |-\alpha,-\alpha\rangle_{4,5} \right] \right. \\
& \left. + |0\rangle_{11} |ODD, 2\alpha\rangle_{12} \left[-\in_+ |-\alpha,-\alpha\rangle_{4,5} - \in_- |\alpha,\alpha\rangle_{4,5} \right] \right\}. \qquad (9)
\end{aligned}
$$

The photon counting result in outputs in modes 11 and 12 is either (*i*) zero in both outputs, or (*ii*) zero in one output and non-zero even in the other, or (*iii*) zero in one output and odd in the other. As we shall see, for (*i*), the teleportation fails, for (*ii*), it is perfect and for (iii), it is almost perfect.

I can write equation (9) in terms of even and odd coherent sates with the help of equation (2). Bob then applies the required unitary transformation and gets the states,

$$|T\rangle_{0,0} = |EVEN, \alpha, \alpha\rangle_{4,5} \; , \qquad (10)$$

$$|T\rangle_{EVEN,0} = |T\rangle_{0,EVEN} = A_+ |ODD, \alpha, \alpha\rangle_{4,5} + A_- |EVEN, \alpha, \alpha\rangle_{4,5} \qquad (11)$$

$$|T\rangle_{ODD,0} = |T\rangle_{0,ODD} \sim A_+ \sqrt{\frac{1-x^4}{1+x^4}} |ODD, \alpha, \alpha\rangle_{4,5} + A_- \sqrt{\frac{1+x^4}{1-x^4}} |EVEN, \alpha, \alpha\rangle_{4,5} \, , \qquad (12)$$

respectively. Sign ~ used in (11) represents the un-normalized states. From (11) we see that the teleportation is perfectly successful with fidelity $\langle T|I\rangle \langle I|T\rangle = 1$ for results (nonzero even, 0) and (0, nonzero even) photon counting. For results (odd, 0) and (0, odd) the fidelity is

$$F_{IV} = F_V \equiv \langle T|I\rangle \langle I|T\rangle = \frac{\left[1 - x^4 \left(|A_+|^2 - |A_-|^2 \right) \right]^2}{1 + x^8 - 2x^4 \left(|A_+|^2 - |A_-|^2 \right)} = \frac{\left(1 - x^4 \cos\theta \right)^2}{1 + x^8 - 2x^4 \cos\theta} \qquad (13)$$

From equation (13) it is clear that maxima are obtained at $\theta = 0$ and π, and a minimum is obtained at $\theta = \cos^{-1}\left(x^4\right)$, and that the MAF $\equiv F_{\min}$ is $\left(1 - x^8\right)$. This is very close to unity for appreciable value of $|\alpha|^2$. When the result of photon counting is zero in both output modes, the teleported state is $|ODD, \alpha\rangle$ which gives maximum fidelity equal to 1 for information $|ODD, \alpha\rangle$ and minimum fidelity equal to zero for information $|EVEN, \alpha\rangle$. MAF is zero in this case but the probability of getting this result is very very small.

I define the average fidelity as the sum of the product of probability for occurrence of a case (zero or non-zero even or odd photon counts) and the corresponding fidelity i.e. $F_{av.} = \sum_{i=I}^{V} P_i F_i$ is $F_{av.} = 1 - \frac{2\, x^2 |A_+|^2 \left[x^4 |A_-|^2 + |A_+|^2 \right]}{(1+x^4)^2}$, which has the minimum value $F_{av.\min} = 1 - \frac{2\, x^4}{(1+x^4)^2}$ for $A_- = 0$. For $|\alpha|^2$=2, the value of $F_{av.\min}$ 0.9999 which is near unity.

2. CONCLUSION

I conclude that the probability of successful teleportation can be made very nearly equal to unity for an appreciable value of $|\alpha|^2$. Wang had, however, concluded that the probability for successful teleportation in this case is only ½. I noted that an almost perfect teleportation can be obtained by changing the scheme of photon counting measurement. Wang separated results for even and odd counts and reported failure for even and success for odd counts. I separated vacuum state from even state and named it as non-zero even state and hence my in my case the photon counting results were divided into three states: zero, non-zero even and odd photon counts. For zero counts we report a failure (MAF=0), but the probability of getting this state is much small. For non-zero even counts, I report perfect success (MAF=1) and for odd count, I report an almost perfect success (MAF$\cong$1) for an appreciable mean number of photons.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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