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Derivation of Minimal Cutsets from Minimal Pathsets for a Multi-State System and Utilization of Both Sets in Checking Reliability Expressions

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Authors' contributions

This work was carried out in collaboration between the two authors. Author AMAR wrote the entire draft of the manuscript, conducted the mathematical and conceptual analyses and managed the basic literature survey. Author MHA participated in the literature search and performed the computational work. Both authors read and approved the final manuscript.

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ABSTRACT

This paper addresses two important useful extensions of binary reliability techniques to multi-state reliability techniques, namely: (a) the problem of complementation or inversion of the function of system success to that of system failure (or equivalently, of deriving the logical minimal cutsets in terms of the logical minimal paths), and (b) the associated problem of hand-checking of a symbolic reliability expression. The paper deals specifically with the reliability of a multi-state delivery network. It presents two complementation procedures, one via the application of multi-state De Morgan's rules, and the other via the multi-state Boole-Shannon expansion. The paper also illustrates one case in which this complementation is needed, as it outlines a method for checking the reliability of the multi-state system in terms of its logical minimal paths and logical minimal cutsets.

Keywords: Network reliability; complementation; De' Morgan laws; Boole-Shannon expansion; symbolic checking; probability-ready expression; multi-state system.

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1. INTRODUCTION

A prominent problem of reliability engineering is the problem of obtaining the minimal sum for a switching function (Two-valued Boolean function) whenever the minimal sum for its complement is given, or equivalently, deriving the prime implicants (PIs) of a function in terms of those of its complement. This problem is usually referred to as the inversion problem, and in reliability context it deals with the derivation of minimal cutsets (PIs of system failure) in terms of minimal paths (PIs of system success) [1-15]. Solution of this problem is necessary when minimal paths are known, while the failure modes or cutsets of the system are needed [4]. Knowledge of both sets of prime implicants for system success and system failure also facilitates system reliability computations [16-18] and is a must for exhaustive checking of symbolic reliability expressions [19, 20].

This paper is a part of an on-going activity [21-32] that strives to provide a pedagogical treatment of multi-state reliability problems, and to establish a clear and insightful interrelationship between the two-state modeling and the multi-state one by stressing that multi-valued concepts are natural and simple extensions of two-valued ones. The paper addresses two important useful extensions of binary techniques to multi-valued techniques, namely: (a) the afore-mentioned problem of complementation of system success to system failure, and (b) the associated problem of hand-checking of symbolic reliability expressions.

As a vehicle for demonstrating the afore-mentioned extensions, we consider a multi-state delivery network (MSDN) with multiple suppliers, in which a vertex denotes a supplier, a transfer station or a market, while a branch denotes a carrier providing the delivery service for a pair of vertices [33]. The capacity that is available for a specific customer of the carrier responsible for the delivery on a branch is treated as a multi-state variable, since this capacity is shared among several customers including the one under consideration. The addressed problem is to evaluate the network reliability, the probability that the MSDN with the deterioration consideration can satisfy the market demand within the budget and production capacity limitations. Lin et al. [33] developed an algorithm, which, among other things, deduced the binary system success in terms of the multi-valued component successes. The logical expression of this success is a disjunctive normal form (DNF) or a sum-of products expression, in which the products are prime implicants that are called (logical) minimal paths. The expected value of this expression is system reliability. Our specific tasks herein are to complement system success to obtain system failure as a disjunction of cutsets, and to hand-check a typical reliability expression for the system. Standard notation of representing multi-state quantities via binary instances are employed throughout this paper [22, 25-30,32].

The organization of the remainder of this paper is as follows. Section 2 presents important pertinent assumptions. Section 3 introduces the running example of a multi-state delivery network (MSDN) with multiple suppliers, borrowed from [33]. Section 3 reproduces from [33] the system success used as a starting point herein. Section 4 presents a complementation procedure via the application of multi-state De Morgan's rules. Section 5 provides an alternative complementation technique via the multi-state Boole-Shannon expansion. Section 6 outlines a method for checking the reliability of the multi-state system in terms of its minimal paths and minimal cutsets. Section 7 concludes the paper.

2. ASSUMPTIONS

The model considered is one of a system with binary output and multistate components, specified by the structure or success function $\mathrm{S(X)}$ [25, 34].

$$\mathrm{S}: \{0, 1, \cdots, \mathrm{m}_1\} \times \{0, 1, \cdots, \mathrm{m}_2\} \times \ldots \times \{0, 1, \cdots, \mathrm{m}_\mathrm{n}\} \rightarrow \{0, 1\}. \quad (1)$$

The system is generally non-homogeneous, i.e., the number of system states (two) and the numbers of component states $(\mathrm{m}_1 + 1), (\mathrm{m}_2 + 1), \cdots, (\mathrm{m}_\mathrm{n} + 1)$ might differ [34]. When these numbers have a common value , the system reduces to a homogeneous one.

The system is a non-repairable one with statistically independent non-identical (heterogeneous) components.
The system is a coherent one enjoying the properties of causality, monotonicity, and component relevancy [21-27].

3. DETAILED RUNNING EXAMPLE

Lin et al. [33] studied the multi-state delivery network (MSDN) with multiple suppliers shown in Fig. 1. The network contains two suppliers, one market, two transfer centers and eight branches.

The network has specific data of delivery costs, probability distributions of all branches and available capacities that are listed in [33], together with the suppliers' production capacities. The physical minimal paths (PMPs) connecting source s_1 and terminal t can be expressed as $P_1 = \{b_1, b_6\}$, $P_2 = \{b_2, b_7\}$ and $P_3 = \{b_2, b_5, b_8\}$, and the PMPs connecting source s_2 and terminal t are $P_4 = \{b_3, b_7\}$, $P_5 = \{b_3, b_5, b_8\}$ and $P_6 = \{b_4, b_8\}$. The deterioration rate vector for the six PMPs is given together with the demand, production capacity and the budget. The final success expression derived from Table in [33] (obtained via lengthy manipulations, and reported here with appropriate translation of notation) is given by the following expression, which is a disjunction of eight prime implicants or logical minimal paths (LMPs) of system success.

$$\begin{aligned} S = {} & X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\} \\ & \vee X_3\{\geq 3\}\, X_7\{\geq 3\} \\ & \vee X_2\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\} \\ & \vee X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\} \\ & \vee X_2\{\geq 3\}\, X_7\{\geq 3\} \\ & \vee X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\} \\ & \vee X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\} \\ & \vee X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 3\}. \end{aligned} \tag{2}$$

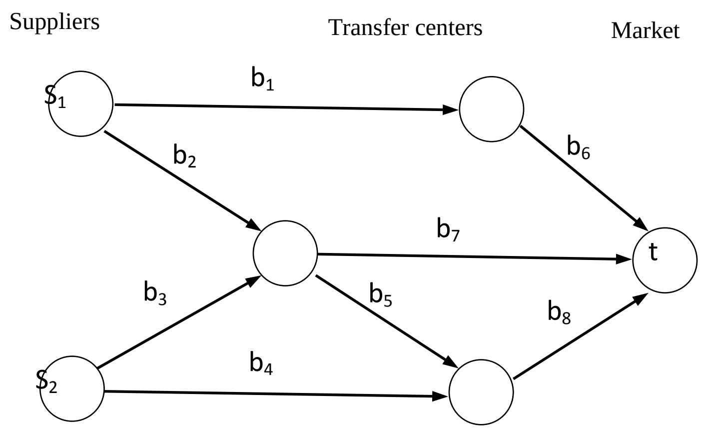

Fig. 1. The multi-state delivery network (MSDN) with multiple suppliers studied in [33]

4. COMPLEMENTATION VIA APPLICATION OF MULTI-STATE DE MORGAN'S RULES

De Morgan's laws are a pair of transformation rules [35-40] that are both valid rules of inference [41-44]. These rules express conjunctions (ANDing) and disjunctions (ORing) solely in terms of each other via negation or complementation as follows: The negation of a disjunction is the conjunction of the negations, while the negation of a conjunction is the disjunction of the negations. This is expressed mathematically as [18]

$$\overline{\vee_{k=1}^{n} A_k} = \wedge_{k=1}^{n} \bar{A}_k, \tag{3}$$

$$\overline{\wedge_{k=1}^{n} A_k} = \vee_{k=1}^{n} \overline{A_k}\,. \tag{4}$$

Implementation of De Morgan's rules (in the multi-state case) necessitates the use of several simplification rules. An important simplification rule it uses (when handling coherent success) is the following domination rule (which generalizes the idempotency rule of AND for an uncomplemented literal ($X_k \wedge X_k = X_k$) in the two-valued case) [32]

$$X_k(\geq j_1)\, X_k(\geq j_2) = X_k(\geq j_2) \qquad \text{for} \quad j_2 \geq j_1, \tag{5a}$$

A similar simplification used by De Morgan's rules (when handling coherent failure) is the following domination rule (which is another generalization of the idempotency rule of AND for a complemented literal ($\bar{X}_k \wedge \bar{X}_k = \bar{X}_k$) in the two-valued case) [32]

$$X_k(\leq j_1)\, X_k(\leq j_2) = X_k(\leq j_2) \qquad \text{for} \quad j_2 \leq j_1, \tag{5b}$$

Two other important simplification rules are the two differencing rules [32]

$$X_k(\geq j_1)\, X_k(\leq j_2) = X_k(j_1, j_1+1, \ldots, j_2) \qquad \text{for} \quad j_2 \geq j_1, \tag{6a}$$

$$X_k(\geq j_1)\, X_k(< j_2) = X_k(j_1, j_1+1, \ldots, j_2-1) \qquad \text{for} \quad j_2 > j_1, \tag{6b}$$

which have no counterpart in the two-valued case, unless they are replaced by the orthogonality rules (which generalize the orthogonality ($X_k \wedge \bar{X}_k = 0$) in the two-valued case)

$$X_k(\geq j_1)\, X_k(\leq j_2) = 0 \qquad \text{for} \quad j_2 < j_1, \tag{6c}$$

$$X_k(\geq j_1)\, X_k(< j_2) = 0 \qquad \text{for} \quad j_2 \leq j_1, \tag{6d}$$

$$X_k(j)\, X_k(\neq j) = 0, \tag{6e}$$

Three other important simplification rules are the three complementation rules [32]

$$\bar{X}_k\{\geq j\} = X_k\{< j\}, \tag{7a}$$

$$\bar{X}_k\{> j\} = X_k\{\leq j\}, \tag{7b}$$

$$\bar{X}_k\{j\} = X_k\{\neq j\}. \tag{7c}$$

As a prelude to the complementation of the coherent success S in our running example, we rearrange the terms of S in (2) to obtain

$$\begin{aligned} S = {} & X_3\{\geq 3\}\, \boldsymbol{X_7}\{\geq \boldsymbol{3}\} \\ & \vee\ X_2\{\geq 3\}\, \boldsymbol{X_7}\{\geq \boldsymbol{3}\} \\ & \vee\ X_3\{\geq 3\}\, \boldsymbol{X_5}\{\geq \boldsymbol{3}\}\, \boldsymbol{X_8}\{\geq \boldsymbol{3}\} \\ & \vee\ X_2\{\geq 3\}\, \boldsymbol{X_5}\{\geq \boldsymbol{3}\}\, \boldsymbol{X_8}\{\geq \boldsymbol{3}\} \\ & \vee\ X_2\{\geq 2\}\, \boldsymbol{X_3}\{\geq 2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\} \\ & \vee\ X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\} \\ & \vee\ X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 3\} \\ & \vee\ X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}\boldsymbol{X_6}\{\geq \boldsymbol{2}\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}\ . \end{aligned} \tag{8}$$

We now combine similar terms to obtain

$$\begin{aligned} S = {} & \boldsymbol{X_7}\{\geq \boldsymbol{3}\}(\, X_2\{\geq 3\} \vee\ X_3\{\geq 3\}) \\ & \vee \boldsymbol{X_5}\{\geq \boldsymbol{3}\}\, \boldsymbol{X_8}\{\geq \boldsymbol{3}\}(\, X_2\{\geq 3\} \vee\ X_3\{\geq 3\}) \\ & \vee\ \boldsymbol{X_3}\{\geq \boldsymbol{2}\}\, \boldsymbol{X_4}\{\geq \boldsymbol{2}\}\, \boldsymbol{X_8}\{\geq \boldsymbol{2}\}(X_2\{\geq 2\}X_7\{\geq 3\} \vee X_1\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}) \\ & \vee\ \boldsymbol{X_1}\{\geq \boldsymbol{2}\}\, \boldsymbol{X_2}\{\geq \boldsymbol{2}\}\, \boldsymbol{X_6}\{\geq \boldsymbol{2}\}\, (\, X_3\{\geq 2\}X_7\{\geq 3\} \vee\ X_4\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}). \end{aligned} \tag{9}$$

Complementing both sides of (9), we obtain the system failure as a product of four sums (which, when multiplied out, and after absorbing all subsuming terms, yields a disjunction of all prime implicants of system failure)

$$\bar{S} = (X_7\{< 3\} \vee\ X_2\{< 3\}\, X_3\{< 3\})\ (X_5\{< 3\} \vee\ X_8\{< 3\} \vee X_2\{< 3\}\, X_3\{< 3\})\ A\, B. \tag{10}$$

Where

$$A = (X_3\{<2\} \vee X_4\{<2\} \vee X_8\{<2\} \vee (X_2\{<2\} \vee X_7\{<3\})(X_1\{<2\} \vee X_6\{<2\} \vee X_7\{<2\})), \quad (11)$$

$$B = (X_1\{<2\} \vee X_2\{<2\} \vee X_6\{<2\} \vee (X_3\{<2\} \vee X_7\{<3\})(X_4\{<2\} \vee X_7\{<2\} \vee X_8\{<2\})). \quad (12)$$

Using intelligent multiplication [40, 42, 44-52] $\{(a \vee b)(a \vee c) = (a \vee bc)\}$, we multiply out the first two terms in (10) to obtain

$$\begin{aligned}&(X_7\{<3\} \vee X_2\{<3\} X_3\{<3\})\ (X_5\{<3\} \vee X_8\{<3\} \vee X_2\{<3\} X_3\{<3\})\\ &= X_7\{<3\}\ (X_5\{<3\} \vee X_8\{<3\}) \vee X_2\{<3\} X_3\{<3\}\\ &= X_7\{<3\} X_5\{<3\} \vee X_7\{<3\}\ X_8\{<3\} \vee X_2\{<3\} X_3\{<3\}\end{aligned}$$

Hence, the expression for $\bar{S}$ can be rewritten as

$$\bar{S} = X_7\{<3\} X_5\{<3\} A B \vee X_7\{<3\}\ X_8\{<3\} A B \vee X_2\{<3\} X_3\{<3\} A B. \quad (10a)$$

The three terms in the expression (10a) above for $\bar{S}$ can be simplified (making use of Boolean quotients [18, 22, 25, 40, 43, 53, 54]) to

$$\begin{aligned}&X_7\{<3\} X_5\{<3\} A B = X_7\{<3\} X_5\{<3\}\ (A B / X_7\{<3\} X_5\{<3\}) = X_7\{<3\} X_5\{<3\}(X_3\{<2\} \vee X_4\{<2\} \vee X_8\{<2\} \vee X_1\{<2\} \vee X_6\{<2\} \vee X_7\{<2\})\ (X_1\{<2\} \vee X_2\{<2\} \vee X_6\{<2\} \vee X_4\{<2\} \vee X_7\{<2\} \vee X_8\{<2\})\\ &= X_7\{<3\} X_5\{<3\}(X_2\{<2\} X_3\{<2\} \vee X_4\{<2\} \vee X_8\{<2\} \vee X_1\{<2\} \vee X_6\{<2\} \vee X_7\{<2\})\\ &= X_7\{<3\} X_5\{<3\}(X_2\{<2\} X_3\{<2\} \vee X_4\{<2\} \vee X_8\{<2\} \vee X_1\{<2\} \vee X_6\{<2\}) \vee X_7\{<2\} X_5\{<3\}\end{aligned}$$

$$\begin{aligned}&X_7\{<3\} X_8\{<3\} A B = X_7\{<3\} X_8\{<3\}\ (A B / X_7\{<3\} X_8\{<3\}) = X_7\{<3\} X_8\{<3\}(X_3\{<2\} \vee X_4\{<2\} \vee X_8\{<2\} \vee X_1\{<2\} \vee X_6\{<2\} \vee X_7\{<2\})\ (X_1\{<2\} \vee X_2\{<2\} \vee X_6\{<2\} \vee X_4\{<2\} \vee X_7\{<2\} \vee X_8\{<2\})\\ &= X_7\{<3\} X_8\{<3\}(X_2\{<2\} X_3\{<2\} \vee X_4\{<2\} \vee X_8\{<2\} \vee X_1\{<2\} \vee X_6\{<2\} \vee X_7\{<2\})\\ &= X_7\{<3\} X_8\{<3\}(X_2\{<2\} X_3\{<2\} \vee X_4\{<2\} \vee X_1\{<2\} \vee X_6\{<2\}) \vee X_7\{<2\} X_8\{<3\}\\ &\vee X_7\{<3\} X_8\{<2\}\end{aligned}$$

$$\begin{aligned}&X_2\{<3\} X_3\{<3\} A B\\ &= X_2\{<3\} X_3\{<3\}\ (X_3\{<2\} \vee X_4\{<2\} \vee X_8\{<2\} \vee (X_2\{<2\} \vee X_7\{<3\})(X_1\{<2\} \vee X_6\{<2\} \vee X_7\{<2\}))\\ &(X_1\{<2\} \vee X_2\{<2\} \vee X_6\{<2\} \vee (X_3\{<2\} \vee X_7\{<3\})(X_4\{<2\} \vee X_7\{<2\} \vee X_8\{<2\}))\\ &= (X_2\{<3\} X_3\{<2\} \vee X_2\{<3\} X_3\{<3\}(X_4\{<2\} \vee X_8\{<2\}) \vee X_3\{<3\}(X_2\{<2\}\\ &\qquad \vee X_2\{<3\} X_7\{<3\})(X_1\{<2\} \vee X_6\{<2\} \vee X_7\{<2\}))\\ &(X_2\{<2\} X_3\{<3\} \vee X_2\{<3\} X_3\{<3\}(X_1\{<2\} \vee X_6\{<2\}) \vee X_2\{<3\}(X_3\{<2\}\\ &\qquad \vee X_3\{<3\} X_7\{<3\})(X_4\{<2\} \vee X_7\{<2\} \vee X_8\{<2\}))\end{aligned}$$

$$\begin{aligned}&= X_2\{<2\} X_3\{<2\} \vee X_2\{<3\} X_3\{<2\}(X_1\{<2\} \vee X_6\{<2\}) \vee X_2\{<3\} X_3\{<2\}(X_4\{<2\} \vee X_7\{<2\} \vee X_8\{<2\})\\ &\vee (X_4\{<2\} \vee X_8\{<2\})\ \ (X_2\{<2\} X_3\{<3\} \vee X_2\{<3\} X_3\{<3\}(X_1\{<2\} \vee X_6\{<2\}) \vee X_2\{<3\}(X_3\{<2\} \vee X_3\{<3\} X_7\{<3\}))\\ &\vee (X_2\{<2\} \vee X_2\{<3\} X_7\{<3\})\ (X_3\{<2\} \vee X_3\{<3\} X_7\{<3\})\ (X_7\{<2\} \vee (X_1\{<2\} \vee X_6\{<2\})\ (X_4\{<2\} \vee X_8\{<2\})\\ &= X_2\{<2\} X_3\{<2\} \vee X_1\{<2\} X_2\{<3\} X_3\{<2\} \vee X_2\{<3\} X_3\{<2\} X_6\{<2\}\\ &\vee X_2\{<3\} X_3\{<2\} X_4\{<2\} \vee X_2\{<3\} X_3\{<2\} X_7\{<2\} \vee X_2\{<3\} X_3\{<2\} X_8\{<2\}\\ &\vee X_2\{<2\} X_3\{<3\} X_4\{<2\} \vee X_2\{<2\} X_3\{<3\} X_8\{<2\}\\ &\vee X_1\{<2\} X_2\{<3\} X_3\{<3\} X_4\{<2\} \vee X_1\{<2\} X_2\{<3\} X_3\{<3\} X_8\{<2\}\\ &\vee X_2\{<3\} X_3\{<3\} X_4\{<2\} X_6\{<2\} \vee X_2\{<3\} X_3\{<3\} X_6\{<2\} X_8\{<2\}\\ &\qquad \vee X_2\{<3\} X_3\{<3\} X_4\{<2\} X_7\{<3\} \vee X_2\{<3\} X_3\{<3\} X_7\{<3\} X_8\{<2\}\end{aligned}$$

The final expression for system failure is given by a disjunction of all 24 logical minimal cutsets (corresponding to the Blake canonical form in the two-valued case) as follows

$$\begin{aligned}\bar{S} = {} & X_4\{<2\}\,X_5\{<3\}\,X_7\{<3\} \vee X_5\{<3\}\,X_7\{<3\}X_8\{<2\} \\ & \vee X_1\{<2\}\,X_5\{<3\}\,X_7\{<3\} \vee X_5\{<3\}\,X_6\{<2\}\,X_7\{<3\} \vee X_5\{<3\}\,X_7\{<2\} \\ & \vee X_1\{<2\}\,X_7\{<3\}\,X_8\{<3\} \vee X_4\{<2\}\,X_7\{<3\}\,X_8\{<3\} \\ & \vee X_6\{<2\}\,X_7\{<3\}\,X_8\{<3\} \vee X_7\{<2\}\,X_8\{<3\} \vee X_7\{<3\}\,X_8\{<2\} \\ & \vee X_2\{<2\}\,X_3\{<2\} \vee X_1\{<2\}\,X_2\{<3\}\,X_3\{<2\} \vee X_2\{<3\}\,X_3\{<2\}\,X_6\{<2\} \\ & \vee X_2\{<3\}\,X_3\{<2\}\,X_4\{<2\} \vee X_2\{<3\}\,X_3\{<2\}\,X_7\{<2\} \vee X_2\{<3\}\,X_3\{<2\}\,X_8\{<2\} \\ & \vee X_2\{<2\}\,X_3\{<3\}\,X_4\{<2\} \vee X_2\{<2\}\,X_3\{<3\}\,X_8\{<2\} \\ & \vee X_1\{<2\}\,X_2\{<3\}\,X_3\{<3\}\,X_4\{<2\} \vee X_1\{<2\}\,X_2\{<3\}\,X_3\{<3\}\,X_8\{<2\} \\ & \vee X_2\{<3\}\,X_3\{<3\}\,X_4\{<2\}\,X_6\{<2\} \vee X_2\{<3\}\,X_3\{<3\}\,X_6\{<2\}\,X_8\{<2\} \\ & \vee X_2\{<3\}\,X_3\{<3\}\,X_4\{<2\}\,X_7\{<3\} \vee X_2\{<3\}\,X_3\{<3\}X_7\{<3\}\,X_8\{<2\}. \end{aligned} \quad (13)$$

5. INVERSION VIA THE BOOLE-SHANNON EXPANSION

A prominent way for converting a Boolean function into its complement is the Boole-Shannon Expansion, which takes the following form in the two-valued case [18, 54-58]

$$f(X) = (\bar{X}_i \wedge f(X|0_i)) \vee (X_i \wedge f(X|1_i)) = \bar{X}_i\ f(X|0_i) \vee X_i\ f(X|1_i), \quad (14)$$

This Boole-Shannon Expansion expresses a (two-valued) Boolean function $f(X)$ in terms of its two sub-functions $f(X|0_i)$ and $f(X|1_i)$ These subfunctions are equal to the Boolean quotients $f(X)/\bar{X}_i$ and $f(X)/X_i$, and hence are obtained by restricting X_i in the expression of $f(X)$ to 0 and 1, respectively. If $f(X)$ is a function of n variables, the two sub-functions $f(X|0_i)$ and $f(X|1_i)$ are functions of at most $(n-1)$ variables. A possible (non-unique) multi-valued extension of (14) is [22, 25, 32]

$$S(X) = X_i\{0\} \wedge (S(X)/X_i\{0\}) \vee X_i\{1\} \wedge (S(X)/X_i\{1\}) \vee X_i\{2\} \wedge (S(X)/X_i\{2\}) \vee X_i\{3\} \wedge (S(X)/X_i\{3\}) \vee \ldots \vee X_i\{m_i\} \wedge (S(X)/X_i\{m_i\}). \quad (15)$$

The expansion (15) serves our purposes very well. Once the sub-functions in (15) are complemented, $S(X)$ will be replaced by its complement $\bar{S}(X)$, namely

$$\bar{S}(X) = X_i\{0\} \wedge (\bar{S}(X)/X_i\{0\}) \vee X_i\{1\} \wedge (\bar{S}(X)/X_i\{1\}) \vee X_i\{2\} \wedge (\bar{S}(X)/X_i\{2\}) \vee X_i\{3\} \wedge (\bar{S}(X)/X_i\{3\}) \vee \ldots \vee X_i\{m_i\} \wedge (\bar{S}(X)/X_i\{m_i\}). \quad (16)$$

We now obtain the Boole-Shannon expansion of system failure $\bar{S}$ with respect to the orthonormal set $\{X_2\{\geq 3\}, X_2\{2\}, X_2\{<2\}\}$, which is given by

$$\bar{S} = X_2\{\geq 3\}\,(\bar{S}/X_2\{\geq 3\}) \vee X_2\{2\}\,(\bar{S}/X_2\{2\}) \vee X_2\{<2\}\,(\bar{S}/X_2\{<2\}). \quad (17)$$

System coherence necessitates that

$$(\bar{S}/X_2\{\geq 3\}) \leq (\bar{S}/X_2\{2\}) \leq (\bar{S}/X_2\{<2\}). \quad (18)$$

and, hence, expression (17) for system failure reduces to

$$\bar{S} = X_2\{\geq 3\}\,(\bar{S}/X_2\{\geq 3\}) \vee X_2\{2\}\,((\bar{S}/X_2\{2\}) \vee (\bar{S}/X_2\{\geq 3\})) \vee X_2\{<2\}\,((\bar{S}/X_2\{<2\}) \vee (\bar{S}/X_2\{2\}) \vee (\bar{S}/X_2\{\geq 3\})). \quad (19a)$$

$$\bar{S} = (X_2\{\geq 3\} \vee X_2\{2\} \vee X_2\{<2\})(\bar{S}/X_2\{\geq 3\}) \vee (X_2\{2\} \vee X_2\{<2\}))\,(\bar{S}/X_2\{2\}) \vee X_2\{<2\}\,(\bar{S}/X_2\{<2\}). \quad (19b)$$

$$\bar{S} = (\bar{S}/X_2\{\geq 3\}) \vee X_2\{<3\}\,(\bar{S}/X_2\{2\}) \vee X_2\{<2\}\,(\bar{S}/X_2\{<2\}). \quad (19c)$$

Where

$$(\bar{S} / X_2\{\geq 3\}) = \text{NOT}(S/ X_2\{\geq 3\}) = \text{NOT}(X_7\{\geq 3\} \vee X_5\{\geq 3\} X_8\{\geq 3\} \vee X_1\{\geq 2\} X_4\{\geq 2\} X_6\{\geq 2\} X_7\{\geq 2\} X_8\{\geq 2\})$$
$$= X_7\{< 3\} (X_5\{< 3\} \vee X_8\{< 3\}) (X_1\{< 2\} \vee X_4\{< 2\} \vee X_6\{< 2\} \vee X_7\{< 2\} \vee X_8\{< 2\})$$
$$= X_1\{< 2\} X_5\{< 3\} X_7\{< 3\} \vee X_4\{< 2\} X_5\{< 3\} X_7\{< 3\} \vee X_6\{< 2\} X_5\{< 3\} X_7\{< 3\} \vee X_5\{< 3\} X_7\{< 2\} \vee X_7\{< 3\} X_5\{< 3\} X_8\{< 2\} \vee X_1\{< 2\} X_7\{< 3\} X_8\{< 3\} \vee X_4\{< 2\} X_7\{< 3\} X_8\{< 3\} \vee X_6\{< 2\} X_7\{< 3\} X_8\{< 3\} \vee X_7\{< 2\} X_8\{< 3\} \vee X_7\{< 3\} X_8\{< 2\}. \quad (20)$$

$$(\bar{S} /X_2\{2\}) = \text{NOT} (S/X_2\{2\}) = \text{NOT}(X_3\{\geq 3\} X_5\{\geq 3\} X_8\{\geq 3\} \vee X_3\{\geq 3\} X_7 \{\geq 3\} \vee 0 \vee X_3\{\geq 2\} X_4\{\geq 2\} X_7\{\geq 3\} X_8\{\geq 2\} \vee 0 \vee X_1\{\geq 2\} X_3\{\geq 2\} X_4\{\geq 2\} X_6\{\geq 2\} X_7\{\geq 2\} X_8\{\geq 2\} \vee X_1\{\geq 2\} X_4\{\geq 2\} X_6\{\geq 2\} X_7\{\geq 2\} X_8\{\geq 2\} \vee X_1\{\geq 2\} X_3\{\geq 2\} X_6\{\geq 2\} X_7\{\geq 3\}) \quad (21)$$

$$(\bar{S} /X_2\{< 2\}) = \text{NOT}(S/X_2\{< 2\}) = \text{NOT}(X_3\{\geq 3\} X_5\{\geq 3\} X_8\{\geq 3\} \vee X_3\{\geq 3\} X_7 \{\geq 3\} \vee 0 \vee 0 \vee 0 \vee X_1\{\geq 2\} X_3\{\geq 2\} X_4\{\geq 2\} X_6\{\geq 2\} X_7\{\geq 2\} X_8\{\geq 2\} \vee 0 \vee 0).$$
$$= (X_3\{< 3\} \vee X_5\{< 3\} \vee X_8\{< 3\}) (X_3\{< 3\} \vee X_7 \{< 3\}) (X_1\{< 2\} \vee X_3\{< 2\} \vee X_4\{< 2\} \vee X_6\{< 2\} \vee X_7\{< 2\} \vee X_8\{< 2\}).$$
$$= (X_3\{< 3\} \vee X_5\{< 3\} X_7 \{< 3\} \vee X_7 \{< 3\} X_8\{< 3\}) (X_1\{< 2\} \vee X_3\{< 2\} \vee X_4\{< 2\} \vee X_6\{< 2\} \vee X_7\{< 2\} \vee X_8\{< 2\}).$$
$$= X_3\{< 2\} \vee X_1\{< 2\} X_3\{< 3\} \vee X_3\{< 3\} X_4\{< 2\} \vee X_3\{< 3\} X_6\{< 2\} \vee X_3\{< 3\} X_7\{< 2\} \vee X_3\{< 3\} X_8\{< 2\} \vee X_1\{< 2\} X_5\{< 3\} X_7 \{< 3\} \vee X_3\{< 2\} X_5\{< 3\} X_7 \{< 3\} \vee X_4\{< 2\} X_5\{< 3\} X_7 \{< 3\} \vee X_5\{< 3\} X_6\{< 2\} X_7 \{< 3\} \vee X_5\{< 3\} X_7\{< 2\} \vee X_1\{< 2\} X_7 \{< 3\} X_8\{< 3\} \vee X_3\{< 2\} X_7 \{< 3\} X_8\{< 3\} \vee X_4\{< 2\} X_7 \{< 3\} X_8\{< 3\} \vee X_6\{< 2\} X_7 \{< 3\} X_8\{< 3\} \vee X_7\{< 2\} X_8\{< 3\} \vee X_7 \{< 3\} X_8\{< 2\}. \quad (22)$$

Substitution of (20)-(22) into (19c), followed by absorption of any subsuming terms, yields a formula for $\bar{S}$ that is similar to (13).

6. CHECKING MULTI-STATE RELIABILITY IN TERMS OF MINIMAL PATHS AND CUTSETS

Rushdi [19] introduced tests for checking a symbolic binary reliability expression. These tests might be easily extended to the multi-state case. What is needed is (a) that the reliability expression be a multi-affine function in each of its arguments (a straight line relation in each of the arguments), where an argument is the expectation of certain instance(s) of some multi-valued variable. Moreover, (b) the reliability expression must have a correct "truth table", i.e., must yield a value of '1' in every success state and a value '0' in every failure state. Requirement (b) is substantially simplified by using a 'reduced truth table', each of whose lines asserts either a logical minimal path or a logical minimal cutset. Simply stated, requirement (b) now asserts that the reliability expression must yield a value of '1' when a logical minimal path is asserted and a value '0' when a logical minimal cutset is asserted. Assertion of a logical minimal path or cutset means that the variable instances appearing in it are asserted, while the variables missing in it are 'indeterminate' or "don't-cares." Note that the reduced truth table in Table 1 exhausts all system states since the disjunction of all logical minimal pathsets (all prime implicants of the Boolean function of success) and all logical minimal cutsets (all prime implicants of the complementary Boolean function of failure) constitutes a disjunction of a Boolean function and its complement, which is identically equal to 1. This reduced truth table differs from a conventional truth table, since the lines of the former table might be overlapping, while those of the latter table are disjoint. The present reduced truth table has only 32 lines (representing 8 logical minimal paths plus 24 logical minimal cutsets), which are significantly fewer than those of the conventional truth table, viz., $m_1 * m_2 * m_3 * m_4 * m_5 * m_6 * m_7 * m_8 = 3 * 4 * 4 * 3 * 4 * 3 * 4 * 4 = 27648$.

For the problem of the running example, Rushdi and Amashah [32] obtained the following probability-ready expression (PRE), in which any ORed entities are disjoint and any ANDed entities are statistically independent [16-18, 21-30]

$$S = X_7\{<2\}\ (X_3\{\geq 3\} \lor X_2\{\geq 3\} X_3\{<3\})\ X_5\{\geq 3\} X_8\{\geq 3\} \lor X_7\{2\}\ (X_8\{2\} (X_1\{\geq 2\} (X_2\{\geq 2\} \lor X_2\{<2\} X_3\{\geq 2\})\ X_4\{\geq 2\} X_6\{\geq 2\}) \lor X_8\{\geq 3\} ((X_2\{\geq 3\} \lor X_2\{<3\} X_3\{\geq 3\})\ (X_5\{\geq 3\} \lor X_1\{\geq 2\} X_4\{\geq 2\} X_5\{<3\} X_6\{\geq 2\}) \lor X_1\{\geq 2\} (X_2\{2\} X_3\{<3\} \lor X_2\{<2\} X_3\{2\})\ X_4\{\geq 2\} X_6\{\geq 2\}) \lor X_7\{\geq 3\}\ (X_2\{\geq 3\} \lor X_2\{<3\} X_3\{\geq 3\} \lor X_2\{2\} X_3\{2\}\ (X_4\{\geq 2\} X_8\{\geq 2\} \lor (X_4\{<2\} \lor X_4\{\geq 2\} X_8\{<2\})\ X_1\{\geq 2\} X_6\{\geq 2\}) \lor X_1\{\geq 2\}\ (X_2\{2\} X_3\{<2\} \lor X_2\{<2\} X_3\{2\})\ X_4\{\geq 2\} X_6\{\geq 2\} X_8\{\geq 2\}). \quad (23)$$

This PRE is converted, on a one-to-one basis, into an expectation, by replacing each Boolean variable and Boolean operator by its arithmetic counterpart, namely

$$E\{S\} = E\{X_7\{<2\}\}\ (E\{X_3\{\geq 3\}\} + E\{X_2\{\geq 3\}\} E\{X_3\{<3\}\})\ E\{X_5\{\geq 3\}\} E\{X_8\{\geq 3\}\} + E\{X_7\{2\}\}\ (E\{X_8\{2\}\} (E\{X_1\{\geq 2\}\} (E\{X_2\{\geq 2\}\} + E\{X_2\{<2\}\} E\{X_3\{\geq 2\}\})\ E\{X_4\{\geq 2\}\} E\{X_6\{\geq 2\}\}) + E\{X_8\{\geq 3\}\}\ (E\{(X_2\{\geq 3\}\} + E\{X_2\{<3\}\} E\{X_3\{\geq 3\}\})\ (E\{X_5\{\geq 3\}\} + E\{X_1\{\geq 2\}\} E\{X_4\{\geq 2\}\} E\{X_5\{<3\}\} E\{X_6\{\geq 2\}\}) + E\{X_1\{\geq 2\}\} (E\{X_2\{2\}\} E\{X_3\{<3\}\} + E\{X_2\{<2\}\} E\{X_3\{2\}\})\ E\{X_4\{\geq 2\}\} E\{X_6\{\geq 2\}\}) + E\{X_7\{\geq 3\}\}\ (E\{X_2\{\geq 3\}\} + E\{X_2\{<3\}\} E\{X_3\{\geq 3\}\} + E\{X_2\{2\}\} E\{X_3\{2\}\}\ (E\{X_4\{\geq 2\}\} E\{X_8\{\geq 2\}\} + (E\{X_4\{<2\}\} + E\{X_4\{\geq 2\}\} E\{X_8\{<2\}\})\ E\{X_1\{\geq 2\}\} E\{X_6\{\geq 2\}\}) + E\{X_1\{\geq 2\}\}\ (E\{X_2\{2\}\} E\{X_3\{<2\}\} + E\{X_2\{<2\}\} E\{X_3\{2\}\})\ E\{X_4\{\geq 2\}\} E\{X_6\{\geq 2\}\} E\{X_8\{\geq 2\}\}). \quad (24)$$

Table 1. Reduced 'truth table' for the multi-state reliability function

Logical Minimal Path or Cutset Asserted	$E\{S\}$
$X_3\{\geq 3\} X_7\{\geq 3\}$	1
$X_2\{\geq 3\} X_7\{\geq 3\}$	1
$X_3\{\geq 3\} X_5\{\geq 3\} X_8\{\geq 3\}$	1
$X_2\{\geq 3\} X_5\{\geq 3\} X_8\{\geq 3\}$	1
$X_2\{\geq 2\} X_3\{\geq 2\} X_4\{\geq 2\} X_7\{\geq 3\} X_8\{\geq 2\}$	1
$X_1\{\geq 2\} X_2\{\geq 2\} X_3\{\geq 2\} X_6\{\geq 2\} X_7\{\geq 3\}$	1
$X_1\{\geq 2\} X_3\{\geq 2\} X_4\{\geq 2\} X_6\{\geq 2\}$	1
$X_7\{\geq 2\} X_8\{\geq 2\}$	
$X_1\{\geq 2\} X_2\{\geq 2\} X_4\{\geq 2\} X_6\{\geq 2\} X_7\{\geq 2\} X_8\{\geq 2\}$	1
$X_4\{<2\} X_5\{<3\} X_7\{<3\}$	0
$X_5\{<3\} X_7\{<3\} X_8\{<2\}$	0
$X_1\{<2\} X_5\{<3\} X_7\{<3\}$	0
$X_5\{<3\} X_6\{<2\} X_7\{<3\}$	0
$X_5\{<3\} X_7\{<2\}$	0
$X_1\{<2\} X_7\{<3\} X_8\{<3\}$	0
$X_4\{<2\} X_7\{<3\} X_8\{<3\}$	0
$X_6\{<2\} X_7\{<3\} X_8\{<3\}$	0
$X_7\{<2\} X_8\{<3\}$	0
$X_7\{<3\} X_8\{<2\}$	0
$X_2\{<2\} X_3\{<2\}$	0
$X_1\{<2\} X_2\{<3\} X_3\{<2\}$	0
$X_2\{<3\} X_3\{<2\} X_6\{<2\}$	0
$X_2\{<3\} X_3\{<2\} X_4\{<2\}$	0
$X_2\{<3\} X_3\{<2\} X_7\{<2\}$	0
$X_2\{<3\} X_3\{<2\} X_8\{<2\}$	0
$X_2\{<2\} X_3\{<3\} X_4\{<2\}$	0
$X_2\{<2\} X_3\{<3\} X_8\{<2\}$	0
$X_1\{<2\} X_2\{<3\} X_3\{<3\} X_4\{<2\}$	0
$X_1\{<2\} X_2\{<3\} X_3\{<3\} X_8\{<2\}$	0
$X_2\{<3\} X_3\{<3\} X_4\{<2\} X_6\{<2\}$	0
$X_2\{<3\} X_3\{<3\} X_6\{<2\} X_8\{<2\}$	0
$X_2\{<3\} X_3\{<3\} X_4\{<2\} X_7\{<3\}$	0
$X_2\{<3\} X_3\{<3\} X_7\{<3\} X_8\{<2\}$	0

This reliability expression is a multi-affine function in each of its arguments, and it has a correct reduced truth table, as shown in Table 1. To understand how Table 1 is constructed, we explain a case of one particular logical minimal path and another for a logical minimal cutset. For the logical minimal path $X_3\{\geq 3\}\,X_7\,\{\geq 3\}$, we substitute $E\{X_3\{\geq 3\}\} = 1$ and $E\{X_7\{\geq 3\}\} = 1$ (and hence $E\{X_3\{< 3\}=0$ and $E\{X7<3\}=0$) in equation (24) to obtain

$$E\{S\} = 0 \;\; + 0 + (1)\,(E\{X_2\{\geq 3\}\} + \; E\{X_2\{< 3\}\}(1) + 0 \; = 1. \qquad (25)$$

For the minimal cutset $X_7\{< 3\}\,X_8\,\{< 2\}$, we substitute $E\{X_7\{< 3\}\} = 1$ and $E\{X_8\{< 2\}\} = 1$ (and hence $E\{X_7\{\geq 3\}\} = 0$ and $E\{X_8\{\geq 2\}\} = 0$) in equation (24) to obtain

$$E\{S\} = \; (1)(0) + (1)(0) + 0 = 0. \qquad (26)$$

7. CONCLUSIONS

This paper clarifies the relation between the logical minimal cutsets and logical minimal paths of a multi-state system. It identifies the logical minimal cutsets as prime implicants of the Boolean function of system failure, and recognizes the logical minimal pathsets as prime implicants of the complementary Boolean function of system success [59-70]. The paper offers two approaches for the inversion problem dealing with the complementation of multi-state success into multi-state failure. The two approaches are also valid for the opposite direction for complementation of multi-state failure into multi-state success. A novel contribution of the paper is a listing of simplification rules that need to be associated with De' Morgan rules for the multi-valued case.

This paper is a part of an on-going activity that strives to provide a pedagogical treatment of multi-state reliability problems, and to establish a clear and insightful interrelationship between the two-state modeling and the multi-state one by stressing that multi-valued concepts are natural and simple extensions of two-valued ones. The paper addresses two important useful extensions of binary techniques to multi-valued techniques, namely the problem of complementation of system success to system failure, and the associated problem of hand-checking of symbolic reliability expressions.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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