Enhancing Compressed Sensing with Graph Structural Constraints: A Novel Approach to Active Learning in Measurement Matrices

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

Original Research Article

ABSTRACT

Compressed sensing on the graph, signals can be approximated by the graph and with the nodes containing information, so compressed sensing can collect information distributed on nodes or links. Also, compressed sensing on the graph becomes important due to the high cost of examining parameters one by one and the unavailability of information on some of them directly in the graph. In this article, by using the idea of active learning and random walking, a method has been introduced to improve the construction of the measurement matrix in the field of the graph, so that information from the graph that is used in the construction of the measurement matrix (assuming that the measurement matrix is underdetermined and non-horizontal) is introduced by the random walk method. They may be missed, identified, and, after observation, inserted into the measurement matrix, resulting in a stronger recovery of the original signal. To test this method,

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firstly, from the data set containing five hundred and ninety as the initial signal, the measurement matrix is constructed with two random walking methods and the proposed method, and the output vector is obtained from it, then the initial thin signal is received with two recovery algorithms, convex optimization and model is recovered and finally calculates the amount of error and the degree of similarity of the four recovered signals compared to the original signal and from their comparison, it is clear that the recovery of the thin signal from the matrix made by the proposed method and the recovery with the convex optimization algorithm has the highest The degree of similarity and the lowest amount of error with the original signal is compared to the other three recovered signals.

Keywords: Compressed sensing; graph structural constraint; measurement matrix; active learning.

1. INTRODUCTION

Compressed sampling or sensing is an emerging research field with applications in signal processing, error correction, medical imaging, seismology, and many other fields. By using the compressed sensing technique, a thin or compressible signal vector can be effectively measured and then recovered through linear measurements with dimensions much smaller than the dimensions of the original signal. Compressed sensing has significantly reduced the sampling and computational cost of signals that have a thin or compressible representation [1]. There are differences between intensive sampling in the graph and normal intensive sampling (in the field of image or sound) that distinguish them from each other. Among the most important differences, we can mention the structure of the measurement matrix. Gaussian random matrix is used to construct the measurement matrix in normal compressed sensing (image or image domain), but the observations we have on the graph are all non-negative coefficients, on the other hand, due to the structure and limitations of the network, such as the absence of a link between two nodes, the absence of nodes In the communication vector or other factors, it is not possible to have any observation [2].

If the signal contains information about nodes, every node that is observed must be on the same path and the graph related to the desired network must be connected. If the information of the edges is displayed by the signal, it is still not possible to have any desired observation even if the graph is complete. Network constraints affect the construction of the measurement matrix. It can be said that the family of measurement matrices that can be used for compressed sensing on the graph is more limited than the matrices that can be used in the field of normally compressed sensing (audio or image) and must be proportional to the structure of the graph (network) [3].

Active learning is a special mode of supervised learning, in which the learner interactively asks the information source, or so-called oracle, questions about data labels, that is, instead of providing a large amount of data with labels (which generally cost a lot to prepare), it is enough. A limited number of samples are labeled according to the learner's choice, the learner chooses a question from all the choices he has for the question and asks it, then stores the result of the question in the labeled data set and performs the learning again based on the learned model. And the level of uncertainty based on this model about the samples asks the oracle again. This process is repeated until a certain number of questions are asked or the error rate is less than a certain limit. A comparison between the modes of actively selecting samples for labeling versus the normal mode shows a lower amount of training samples required and a higher accuracy. This mode is especially useful for those cases where training data preparation is expensive and we have a small amount of training data [4-6]. Considering that limited works have been done in the field of measurement matrix construction in the field of compressed sensing on the graph, this article, with an idea similar to the idea of active learning, a method to improve the construction of the measurement matrix has been presented, the result of which is a stronger recovery of the thinnest signal that matches the measurement.

2. LITERATURE REVIEW

A paper titled "Recovery of Sparse Signals Using Markovian Random Field", extended the theory of compressed sensing to include signals that are sparsely represented based on a graphical model. They used Markeffian random fields (MRF) to express and represent signal sparseness, whose non-zero coefficients were grouped or clustered. From the new model-based recovery algorithm known as LAMP Lattice Matching Pursuit (LAMP), they were able

to consistently recover MRF-modeled signals using measurements and calculations far less than the current advanced algorithms [7].

Another article, presented and introduced a new theory in an article titled 'Model-Based Compressed Sensing'. By introducing a theory, they developed model-based compressed sensing that is parallel to conventional theory and provided important guidelines on how to create structured signal recovery algorithms with provable performance guarantees. By reducing the number of degrees of freedom of a thin/compressible signal, they identified two advantages for compressed sensing. First, these instructions enabled them to reduce the number of measurements m needed to recover the signal constant. Second, during signal retrieval, it enabled them to distinguish correct signal information better than other recycled false information, leading to stronger retrieval. To quantify the advantages of model-based compressed sensing, they introduced and studied several new theoretical concepts that could be of general interest. They first introduced structured thin models for K-thin signals. Then, using the model-based RIP property, they proved that these thinly structured signals can be powerfully extracted from noisy compressed measurements. Furthermore, they determined the required number of sizes M and showed that for a number of structured scattering models, M is independent of N. These results generalize the limited works related to structure scattering models for thin fine signals. They then introduced the idea of structured compressible signals. To prove that structured compressible signals can be robustly recovered from compressed sizes, they extended the standard RIP to a new limited amplification property or RAMP. Using RAMP, they showed that the required number of measurements M to recover compressible signals is independent of N. For the practical use of this new theory, they recovered the way of integration, structured thin models with two CoSaMP and iterative hard thresholding (IHT) compressed sensing recovery algorithms. The key modification made was very simple: they simply replaced the nonlinear scatter approximation step in these greedy algorithms with a structured scatter approximation. Now, thanks to this new theory, both model-based recovery algorithms are highly guaranteed to recover structured and sparsely structured compressible signals. To confirm their theory and algorithms and the usefulness of the theory, they presented two specific cases of model-based

compressed sensing and performed simulation experiments [8].

Another article studied the active learning of open global graphs for node classification. The great power of Graph Neural Networks (GNN) relies on a large amount of labeled training data, but obtaining labels can be expensive in many cases. Graph Active Learning (GAL) has been proposed to reduce such annotation costs, but existing methods mainly focus on improving labeling efficiency with fixed classes and are limited to handling the emergence of new classes. This problem was called Open Global Graph Active Learning (OWGAL) and a framework with the same name is proposed in this paper. The key is to identify new as well as instructive class nodes in an integrated framework. Instead of a fully connected neural network classifier, OWGAL uses prototype learning and label propagation to assign high uncertainty scores to target nodes in the representation space and topology. Weighted sampling reduces the influence of insignificant classes by weighing the importance of node and class. Experimental results on four large-scale data sets show that the framework of this paper achieves a significant improvement from 5.97% to 16.57% in Macro-F1 compared to advanced methods [9].

In another study, the teaching of graph representation and its applications was considered. Learning graph representation is an important task because it can facilitate various downstream tasks, such as node classification, link prediction, etc. The goal of graph representation learning is to map graph entities into low-dimensional vectors while preserving the graph structure and entity relationships. Over the decades, many models for graph representation learning have been proposed. The purpose of this article is to show a comprehensive picture of graph representation learning models, including traditional and advanced models on different graphs in different geometric spaces. The authors of this paper first start with five types of graph embedding models: graph kernels, matrix factorization models, shallow models, deep learning models, and non-Euclidean models. In addition, graph transformer models and Gaussian embedding models were also discussed. Then, practical applications of graph embedding models are presented, from constructing graphs for specific domains to using models to solve tasks. Finally, the challenges of existing models and future research directions

were discussed in detail. As a result, this paper presents a structured overview of the variety of graph embedding models [10].

Lal et al. (2023) present a comprehensive review of compressed sensing (CS) techniques specifically tailored for physiological signals. Their analysis, published in the IEEE Sensors Journal, highlights the challenges and innovations in acquiring high-dimensional physiological data efficiently. The authors emphasize the importance of CS in mitigating issues related to data acquisition and storage, which are especially pertinent in healthcare settings. The review categorizes various CS strategies, assessing their effectiveness in different physiological contexts, such as ECG and EEG signal processing. The authors conclude that future research should focus on integrating machine learning algorithms with CS techniques to enhance signal reconstruction accuracy [11].

Wang, Gao, and Xu (2024) extend the discourse around compressed sensing through their study on opportunistic sensing in task-oriented wireless sensor networks. Their work, published in the IEEE Transactions on Network Science and Engineering, explores how graph-based compressed sensing can optimize the performance of sensor networks by reducing energy consumption and improving data collection efficiency. The authors propose a novel framework that combines CS with opportunistic sensing, demonstrating the potential for enhanced network lifetimes and data quality, thereby addressing real-time sensing challenges [12].

In the realm of active learning, Brown et al. (2023) investigate the application of contrastive learning within graph-based active learning frameworks for Synthetic Aperture Radar (SAR) data. Their work, presented at SPIE, highlights how leveraging the structural information inherent in graphs can improve the labeling efficiency of SAR datasets. They demonstrate that contrastive learning can effectively reduce the amount of labeled data required for model training, enhancing performance in object detection tasks. This approach reflects a growing trend in machine learning practices, where unsupervised learning principles are harnessed to augment supervised learning techniques [13].

Simultaneously, Miller and Bertozzi (2024) contribute to the active learning dialogue by focusing on model change scenarios in graph-based semi-supervised learning frameworks. Their paper, published in Communications on Applied Mathematics and Computation, delves into adaptive strategies for model adjustment based on changing data distributions. The authors argue that incorporating model change detection into active learning can lead to more robust and adaptable learning systems, particularly in dynamic environments where data characteristics evolve over time. Their research underscores the necessity of responsive learning models in the face of variability within datasets [14].

3. METHODOLOGY

According to the proposed method, the idea of active learning is applied to the problem of signal-based compressed sensing (active compressed sensing). Considering that the investigated signal describes the characteristics of a network, in this sense the proposed method is a fundamental step in the three fields of compressed sensing, graph, and machine learning. A walker has two basic steps, choosing the starting point of the walk and taking a random walk that follows a connected path. The proposed idea tries to make an intelligent way to choose the starting point. The goal is to predict the ambiguity that we have about each part of the original vector by using the previous observation and starting the observation from a place where the ambiguity is reduced. In each random walk, one row of the measurement matrix is filled, according to the essence of the problem, the values of the matrix of this matrix are zero and one, that is, either an edge is present in an observation (one) or it is not present (zero). The columns of this matrix also correspond to the edges of the network and generally correspond to the main signal channels. The important point is that the sum on a column shows the number of presence of an edge in the observations.

To measure the ambiguity of the measurement matrix, it is suggested to use the presence of an edge in the observations. Based on this criterion, the edge whose corresponding column has the lowest sum is selected. In this way, in each step, an edge is selected to start walking, which has more ambiguity (column sum corresponding to zero edges). If we pay attention to the similarity of this idea with active learning, perhaps another criterion for measuring ambiguity seems to be signal recovery and using the entropy of $p(y|x)$

distribution, but this idea faces two problems. Probabilities are not efficient because the probability distribution p(y|x) will not be available and secondly, it has a very high computational load because it needs to perform a recovery operation once for each observation. In this regard, a criterion is defined regarding the edges of the graph called the awareness criterion (ambiguity image) and it is equal to the number of times that an edge has participated in the observations. To determine the value of this criterion, we assume that we have a large number of observations, but we have not yet observed edge e. The proposed method identifies this edge by calculating the ambiguity criterion and goes to it. However, the normal method works randomly and may not reach this edge.

This criterion is applied to the observation matrix created for the graph. Each row of the measurement matrix corresponds to one observation (step). In each execution of steps, the edges of the graph that are observed are inserted in the corresponding line. Therefore, the number of rows of the measurement matrix is equal to the steps performed on the graph, and the number of ones in each row of the matrix is also equal to the number of edges observed in one step (the sum of the number of ones in each column corresponds to the number of passes through that edge). In calculating the ambiguity measure, we measure the ambiguity of all edges once every time we want to walk. For this, we consider a total row as follows for the observation matrix.

Each row of this matrix contains the set of elements of the corresponding column in the matrix. As the number of steps increases, the matrix and ambiguity criterion will change as follows.

Finally, the matrix and the ambiguity criterion after the third step will be as follows, as can be seen, the ambiguity criterion randomly selects an edge from the set of edges with the lowest number of observations and travels a random path as an observation, and from the equation (1) is obtained.

$$h_i = \sum_{j=1}^{\#RW} a_{ij} \qquad (1)$$

a=random measurement matrix

h= is the sum for the j*j*-th column, and these sums can be computed in parallel for efficiency.

After building the measurement matrix, it is time to recover the signal. Various methods have been provided to recover the signal. The optimization relation related to L1 is in the form of equation (2).

$$\text{Min } |x|_1, y=Ax \qquad (2)$$

Table 1. Measurement matrix

Matrix	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
Sum	0	0	0	0	0	0	0	0	0

Table 2. Observation matrix

Matrix	1	0	0	1	1	1	0	0	0
Sum	1	0	0	1	1	1	0	0	0

Table 3. Matrix and ambiguity criterion

Matrix	1	0	0	1	1	1	0	0	0
	0	1	1	1	1	0	0	0	1
Sum	1	1	1	2	2	1	0	0	1

Table 4. Final observation matrix

Matrix	**1**	**0**	**0**	**1**	**1**	**1**	**0**	**0**	**0**
	0	1	1	1	1	0	0	0	1
	1	0	0	1	0	0	1	1	0
Sum	2	1	1	3	2	1	1	1	1

The main idea in L1 is to use a soft one as a convex approximation of soft zero. In the Ising model, the relationship between the input and output of the recovery system is simulated using a graphical model. In the meantime, a series of new variables are defined for signal attenuation modeling, which are the main signal distribution parameters. These variables are binary and show the zero or non-zero state of a part of the signal, so the number of non-zero elements of the signal is equal to the sum of these variables. First, different degrees of sparseness (k-rate = .01, .001, .005, .05) were applied to the dataset, containing flight information from five hundred airports in the United States of America, to generate the flight delay vector with the specified sparseness rates created (input signal: thin vector x as flight delay vector). Next, the observation matrix was made by the usual method and the proposed method, and then it was recovered by two methods, L1 and Modeling, and finally, the x^ obtained with the initial thin signal x was evaluated separately with the following criteria. The error metric reports the reconstruction error. This measure shows the soft distance between the two original signals and the recovered signal in a normalized and logarithmic scale. The next criterion is the similarity, which shows the power (energy) of the original signal to the noise in a logarithmic scale.

The ambiguity criterion in the proposed active compressed sensing method introduces a mechanism for selecting edges from a graph representation of the signal. By randomly selecting an edge from the set of edges, the method aims to enhance the robustness and adaptability of the signal acquisition process. This random selection serves several important purposes:

1. **Exploration of Signal Space**: Randomly selecting edges allows the method to explore different parts of the signal space, ensuring that various signal characteristics are captured. This exploration is crucial in scenarios where the signal may exhibit sparse representations across different regions.
2. **Reduction of Bias**: By employing a random selection process, the ambiguity criterion mitigates potential biases that could arise from deterministic selection methods. This randomness helps in obtaining a more representative sample of

the signal, leading to improved reconstruction accuracy.
3. **Adaptive Learning**: The random edge selection aligns with the principles of active learning, where the method can adaptively focus on edges that provide the most information about the signal. This adaptability enhances the efficiency of the measurement process, allowing for better utilization of resources.
4. **Parallel Processing**: The random selection of edges can be executed in parallel, similar to the column sum calculations of the measurement matrix. This parallelism not only speeds up the computation but also allows for simultaneous assessments of multiple edges, further optimizing the overall process.
5. **Robustness to Noise**: Randomly selecting edges can also contribute to the robustness of the method against noise and uncertainties in the signal. By diversifying the selection process, the method can better handle variations and maintain performance even in challenging conditions.

4. RESULTS AND DISCUSSION

The description of the proposed method was implemented on the data set consisting of 590. This dataset contains the United States flight network that models the connectivity between airports. This data set contains three columns that specify the airport of origin, destination, and connecting edge number. This graph is undirected and unweighted. To simulate the proposed method, a random delay was created corresponding to each edge of this graph with the assumption that a small number of all edges have a non-zero delay. This generated delay vector is compared with the value generated after observing and recovering the signal. The recovery results are evaluated with two criteria. The error metric reports the reconstruction error. This measure shows the soft distance between the two original signals and the recovered signal in a normalized and logarithmic scale. The second criterion is the SNR criterion, which shows the power (energy) of the main signal to the noise in a logarithmic scale. In drawing the graphs, for easier display, the degree of thinness is also shown in a logarithmic scale. A summary of the real data set and the data set resulting from the implementation of the conventional

method and the proposed method are given separately in Table 1.

The result data set includes two data sets based on the similarity criterion and the Euclidean distance criterion, each of which is approximated separately with thinning rates of .01, .05, .001, .005. A number of data sets of the results with a thinning rate of .001 are shown in Table 2.

This test was performed four times with a thinning rate of 0.01, 0.05, 0.001, and 0.005

separately, and each test was repeated five times, and the results are shown in Figs. 1 to 3.
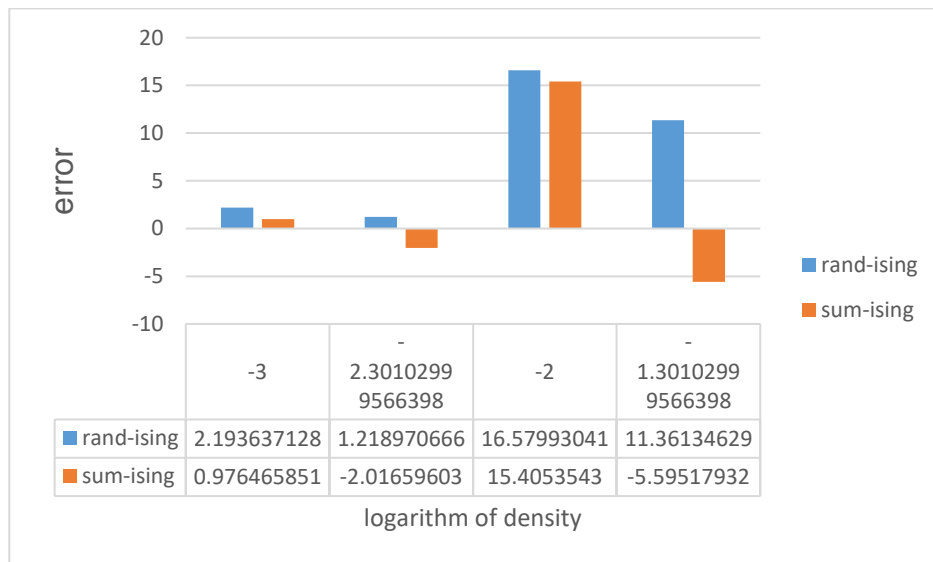
In the above diagram, the proposed method (called sum-ising) and the previous method (called rand-ising) are compared. The horizontal axis shows the logarithm of density (thin image) in base ten and the vertical axis indicates the error which is obtained from equation (3).

$$error=(norm(edge\_delay-X,2)/norm(edge\_delay,2) \quad (3)$$

Table 5. Contains several real data sets

Airport of origin	Destination airport	Connector edge number
2	81	283362
2	165	132110
2	91	104566
2	117	117412
3	258	90464
3	49	368022
3	273	81277

	-3	-2.30102999566398	-2	-1.30102999566398
rand-ising	2.193637128	1.218970666	16.57993041	11.36134629
sum-ising	0.976465851	-2.01659603	15.4053543	-5.59517932

logarithm of density

Fig. 1. Comparison of the proposed sum-ising method and the previous rand-ising method

Table 6. Some of the results of building the measurement matrix by random walker and the proposed method and then recovering the x vector with two L1 algorithms and the Ising model with a thinning rate of 0.001 for the initial vector

		Reconstructed vectors for rate=0.001			
	rand-ising	rand-l1	sum-ising	sum-l1	X
Error	0.0000574990	0.0000980799	0.0000378900	0.0000021715	1.0000000000000000000000
	0.0000000000	0.0001789093	0.0000250605	0.0000000000	0.0000000000000000000000
	0.0000076227	0.0000000000	0.0000000000	0.0000000000	0.0000000000000000000000

At the bottom of the diagram in Fig. (1), the detailed information of each state is shown. To fill the houses of this table and its corresponding diagram, the desired methods have been implemented five times and the average results have been reported. According to this diagram, it can be seen that the amount of error increases with the increase in the density of vectors and moves away from the assumption of thinness. The above diagram is for the case where the recovery method is the Ising model. For the case where the L1 method is used, the following diagram is obtained. In this graph, it can be seen that the error increases with the decrease of the thinness, and it is also seen that in all these cases, the proposed method shows less error than the previous method. So it can be said that the thinner the initial signal is, the recovery error based on the measurement matrix made by the proposed method, and the random walk method, with the convex optimization algorithm, has a lower value than the recovery with the Ising model algorithm. The general result of Fig. (1) is as follows:

✓ Error rate: The recovery error with the L1+ matrix construction algorithm by the proposed method is much less than the recovery error with the L1+ matrix algorithm using the random walk method.
✓ The recovery error with the Ising model algorithm + the proposed method is much less than the recovery with the Ising + matrix by random walk method.

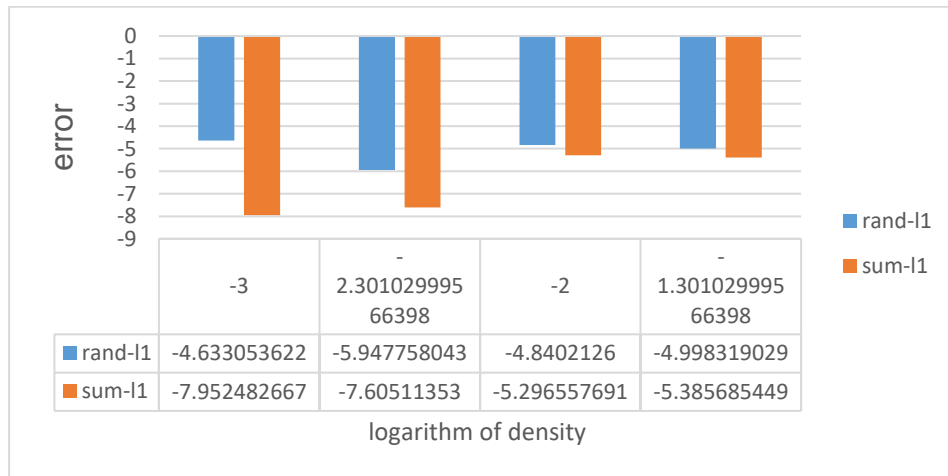In Fig. (2), the error rate diagram is shown in terms of thinness.

	-3	-2.30102999566398	-2	-1.30102999566398
rand-l1	-4.633053622	-5.947758043	-4.8402126	-4.998319029
sum-l1	-7.952482667	-7.60511353	-5.296557691	-5.385685449

logarithm of density

Fig. 2. Error rate diagram in terms of thinness

	-3	-2.30102999566398	-2	-1.30102999566398
rand-ising	-43.8931569	-24.40275077	-331.8374938	-227.3482354
sum-ising	-19.54973155	39.87648607	-312.1909501	111.2483659

logarithm of density

Fig. 3. Graph of the degree of similarity in terms of thinness

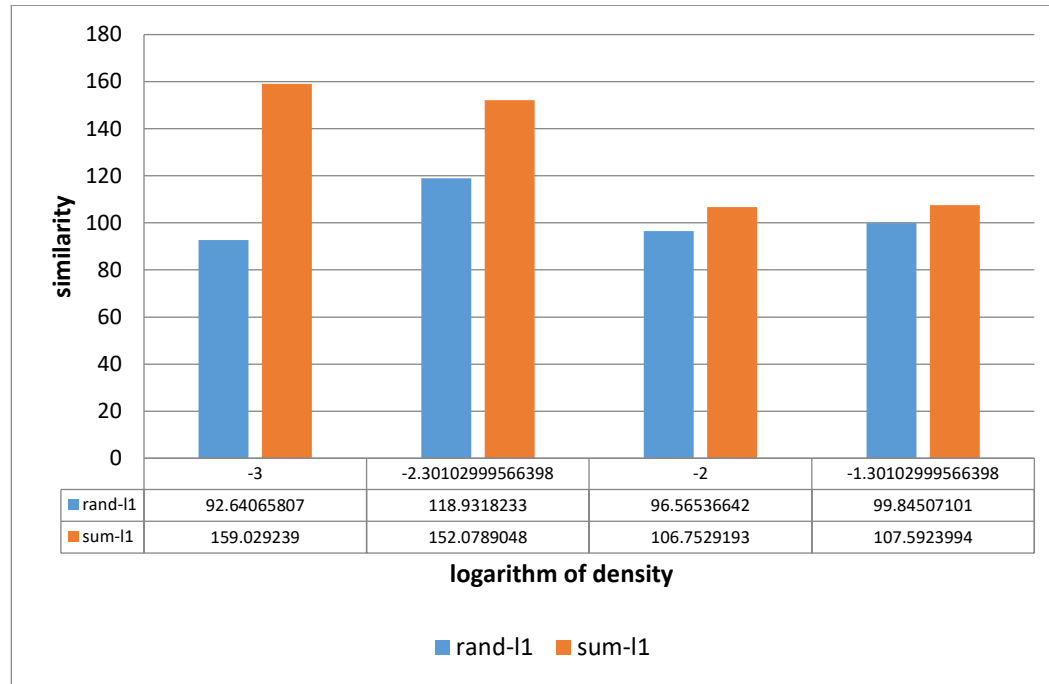|	-3	-2.30102999566398	-2	-1.30102999566398
rand-l1	92.64065807	118.9318233	96.56536642	99.84507101
sum-l1	159.029239	152.0789048	106.7529193	107.5923994

logarithm of density

■ rand-l1   ■ sum-l1
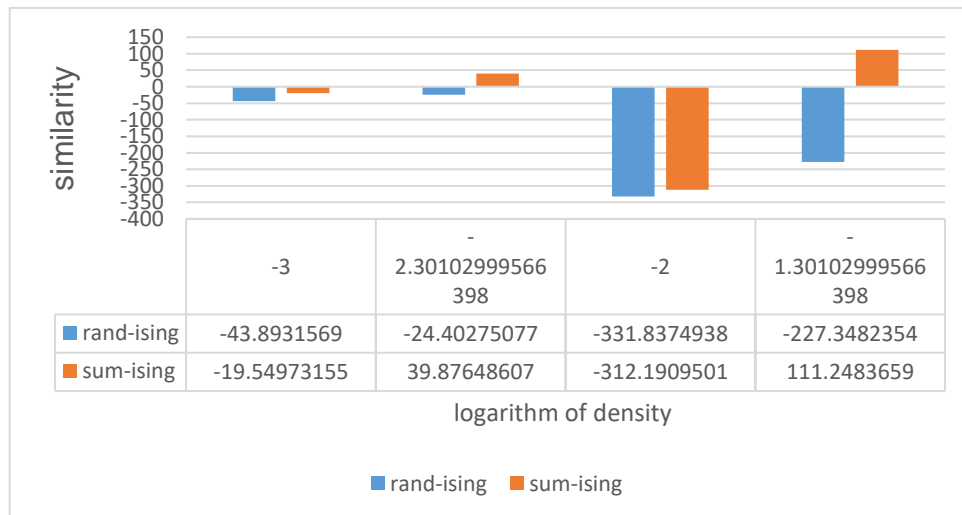
Fig. 4. Graph of the degree of similarity in terms of thinness

The diagram in Figs. (3) and (4) calculates the similarity of the reconstructed vector with the original vector based on the SNR criterion. It can be seen that the SNR of the proposed method is always higher than the competitor method.

When we recover the information collection method using the L1 method and use the SNR criterion to compare the degree of similarity with the original vector, the above graph is obtained, which shows the superiority of the proposed method. As can be seen in this figure, by increasing the signal thinness, the proposed method produces much better results. By examining these four tests, we can say that:

1. The proposed method is generally better than the conventional method.
2. The smaller the thinning rate of the signal (the thinner the input vector), the better the results of the proposed method than the competitor method.

The justification for the proposed method in active compressed sensing can be articulated as follows:

Compressed sensing fundamentally relies on the premise that signals can be represented in a sparse manner, where a significant portion of their elements are zero. This sparsity allows for the reconstruction of signals from fewer measurements than traditionally required. However, the challenge arises in identifying the appropriate sparse representation space, which necessitates complete signal acquisition—a process that contradicts the very essence of compressed sensing, which aims to minimize data acquisition.

To address this conflict, the proposed method employs a random matrix for signal reception. This approach leverages the inherent properties of random matrices, which have been shown to effectively capture the essential features of signals in a compact form with high probability. By utilizing randomness, the method circumvents the need for full signal reception while still enabling accurate reconstruction. It is important to note that while the proposed method incurs a higher computational burden due to the necessity of calculating the sum of the columns of the measurement matrix at each step—serving as an ambiguity criterion—this computation can be efficiently parallelized. Each column's calculations are independent, allowing for significant reductions in processing time through concurrent execution. In summary, the proposed active compressed sensing method effectively balances the need for accurate signal reconstruction with the constraints of data acquisition. By integrating random matrices and active learning principles, it not only enhances accuracy but also maintains computational feasibility through parallel processing, marking a significant advancement in the field of compressed sensing.

5. CONCLUSION

In compressed sensing, it is assumed that the signal to be received has a thin representation in a space, that is, most of its elements are zero. Creating and finding such a space requires a lot of calculations and the signal must be fully received first, but receiving the signal completely conflicts with the purpose of compressed sensing. As a result, a random matrix is used to receive the signal, which has a high probability of receiving the signal in a compact form. In short, the proposed method has applied the idea of active learning in the field of compressed sensing on the graph, and hence it can be called active compressed sensing. The accuracy of the proposed method is higher than the previous method. The proposed method has more computational burden than the previous method because the sum of the columns of the measurement matrix must be calculated in each step to be used as an ambiguity criterion. Of course, these calculations are independent and can be calculated for each column in parallel.

DISCLAIMER (ARTIFICIAL INTELLIGENCE)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of this manuscript.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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