

Assessment of Reaeration Equations for River Tungabhadra, Karnataka, India and Generation of the Refined Equation

**S. Ranjith^{1*}, Anand. V. Shivapur¹, P. Shiva Keshava Kumar²
and Chandrashekarayya. G. Hiremath³**

¹VTU-PG Studies, Belagavi-590018, India.

²Department of Civil Engineering, PDIT Engineering College Hosapete-583201, India.

³Department of Water and Land Management, VTU, Belagavi-590018, India.

Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JGEESI/2019/v21i430130

Editor(s):

(1) Dr. Wen-Cheng Liu, Department of Civil and Disaster Prevention Engineering, National United University, Taiwan and Taiwan Typhoon and Flood Research Institute, National United University, Taipei, Taiwan.

Reviewers:

(1) J. Dario Aristizabal-Ochoa, National University of Colombia, Colombia.

(2) Moses Mwajar Ngeiywa, University of Eldoret, Kenya.

(3) Yongchun zhu, Shenyang Normal University, China.

Complete Peer review History: <http://www.sdiarticle3.com/review-history/49251>

Original Research Article

Received 10 March 2019

Accepted 23 May 2019

Published 10 June 2019

ABSTRACT

The ability of a water body to self-purify itself is dependent on its re-aeration rate (K_a). This rate is necessary to calculate the dissolved oxygen content in the waterbody. This rate also depends on some variables that include the stream velocity, stream bed slope, cross section area, water depth, frictional velocity, discharge rate, Froude's number and a number of other things. For the purpose of this study, thirteen empirical equations are considered when evaluating the performance of the re-aeration rates. This is done with respect to the size of the Tungabhadra river. Observation of the re-aeration rate for this study was done using mass balance approach. The data needed for this was gotten from field investigation data obtained from 288 separate samples (6 different sites) between the period March, 2017 to December, 2018. The performance evaluation of the re-aeration equation was done via the implementation of least square techniques. The following statistical error

*Corresponding author: E-mail: Ranjith.s009@gmail.com;

methods were applied in due course; standard error, normal mean method and mean multiplicative method. The results of the methods are 0.16, -0.0006 and 2.75. The coefficient of correlation for this was 0.91 and by interpretation, it shows an efficient outcome.

Keywords: Dissolved oxygen; DOBT; reaeration coefficient; Tungabhadra River.

Orcid I'd: <https://orcid.org/0000-0002-5342-9842>

1. INTRODUCTION

The mechanism of dissolved oxygen transfer through internal turbulence and mixing has gained much attention in recent years attributing to further study and investigations. Reaeration is the process of physical transfer of oxygen from the atmosphere into the water body, when the concentration of dissolved oxygen goes down with respect to saturation Dissolved oxygen of the stream at a given temperature. In 1925 Streeter and Phelps [1] stated that "Rate of absorption of oxygen is directly proportional to a dissolved oxygen deficit." In 2005 Jain and Jha [2] carried out research work on sensitivity analysis between Dissolved oxygen and Reaeration rate (K_a) whereby they concluded that a small change in K_a gives a larger gap in the Dissolved oxygen. Hence K_a plays a very important role to keep up (maintain) healthy ecosystem of the stream.

Gases gets transfer into water bodies from atmosphere. There are two theories which explain widely for both surface and estuaries water bodies [2-5]. First theory explains only about the standing water and second theory which gives information about the running water. Two film resistance theory assumes that substance moving in a layer by layer form develops maximum resistance between these two layers where the transfer of natural gas takes place. Second theory, i.e. surface renewal model, which assumes stream consisting of layers of water and when these layers are brought to the surface for a period of time, air exchange takes place. As these layers move away from the surface, they mix with the bulk liquid. Prior to the situation envisioned by two-film theory, the dissolved gas penetrates the film, and hence, it is dubbed as penetration theory. In 1951 P.V.Danckwerts [6] altered the elegance by arrogumence that the liquid elements reach and leave the interface arbitrarily and their contact is designated by a statistical delivery. This approach is labeled as the surface renewal theory.

The General Governing equation for oxygen transfer can be written as

$$V (dc/ dt) = K_1 A_s (C_s - C) \quad (1)$$

Where A_s is the surface area of water body (m^2), V is the volume of water body (m^3), K_1 is the mass transfer velocity in liquid laminar layer (md^{-1}), C is the oxygen concentration in water (mgL^{-1}) and C_s is the saturation concentration of oxygen ($mg L^{-1}$).

In cases where the air-water interface is not constricted, the volume is $V = A_s H$, where H is the mean depth (m). Thus, Equation (1) is expressed as

$$(dc/ dt) = K_a (C_s - C) \quad (2)$$

where K_a is the re-aeration rate coefficient (d^{-1}), which is equivalent to $K_a = k_1/H$.

Above equations deliver idea into how the mechanism of K_a operates. The direction and scale of the mass transfer depend on the difference between the saturation value and the actual value of dissolved oxygen concentration in the water. Oxygen re-aeration rate can be induced to different temperatures.

$$K_a (T) = K_a (20) \theta^{T-20} \quad (3)$$

where, $\theta = 1.024$ for pure water. In the rest of this paper, $K_a (20^{\circ})$ is used as K_a .

Many researchers have developed/predicted reaeration equation for both standing and running water. In that 1925 Streeter and Phelps [1] developed a water quality model which became the bible for all researcher to development reaeration equation. William and Cornnor developed empirical equation re-aeration K_a in 1956 based on surface renewal concept which depends on dissolved oxygen balance technique [8,9,10], distribution equilibrium concept [11,12] and tracer method [13] according to researcher K_a are directly proportional to velocity and depth of water. Churchill [9], Dobbing [14] and Streeter and Phelps [1] carried work on experimental esteems. Edward [15], Moog [22] worked on equilibrium distributed technique. Tesivogal and Kernel [19] developed an empirical equation based on tracer techniques. Several empirical

equations developed by using different method but all are related to stream variable such as bed gradient, wetted perimeter, flow velocity, depth of waterway, Froude number, shear stress.

Study area: In this case study we have selected River Tungabhadra which flows through Harihara taluk, Davanagere district, Karnataka, India. In this region stream is heavily polluted by industrial activity and domestic waste discharge at downstream side at Harihara. Tunga and bhadra river are tributaries of Tungabhadra formed by confluence at Koodli at altitude of 610mt above MSL and Tungabhadra travelling along Karnataka and Andhra Pradesh and finally join in River Krishna. Harihara region fall under semi-arid condition in which moderate to higher summer with erratic rainfall and moderate winter with erratic rainfall.

For the present study three villages which are located in the downstream side of Harihara town, namely Nalawagal, Nadiharalahalli, Airani were selected. These places have some socio economic and industrial. The municipal water from all these villages are places are directly discharged into stream and Harihara Poly Fiber which produce rayon grade pulp discharge approximately 30000/- liter per day and Rayon industry which discharge 10000/- liter per day [27]. Both industries are located in left bank of river Tungabhadra near Kumarapattanam. Selection of sampling station was done based on maximum mixing of effluent along width and depth of stream taken. Segment of sampling stations are in shown above Fig. 1.

Calculation of Reaeration constant via DOBT method: As presented in the Fig. 2, the domestic wastewater and industrial effluent are discharged at the segment 2 (just downstream of Harihara taluk) of the stretch. Right at the downstream of the waste discharge point, stretch is subjected to higher BOD concentration. At this point the DO of the stretch exhibits the exponential depletion attributing to the microbial activity in degrading organic matter. Further downstream of the stream, the lower removal rates happen as the more hard-headed natural matter debases at a slower rate and finally Biochemical oxygen demand decay model is shown in equation (4).

$$L=L_0 e^{-kr(x/U)} \quad (4)$$

Streeter and Phelps [1] model derived for a point source of BOD is given by:

$$D=D_0 e^{-Ka(x/u)}+[k_d L_0/(K_a-K_r)][e^{-kr(x/u)}-e^{-ka(x/u)}] \quad (5)$$

where $D=U/S$ Dissolved oxygen deficit $mg.L^{-1}$, X =distance travelled between U/S and D/S in Km, U =stream velocity Km/d, X/U =travel time, L =BOD concentration ($mg.L^{-1}$), L_0 =initial BOD Concentration ($mg.L^{-1}$), K_r =BOD loss rate (d^{-1}), K_d =deoxygenation constant in stream (d^{-1}), K_s =settling removal rate (d^{-1}), $K_r=K_d+K_s$ (d^{-1}) and K_a =reaeration constant (d^{-1}).

For calculation of kinematics constant such as BOD decay rate, Deoxygenation rate, and Reaeration rate, we have selected stretch between 19.5km to 33 km i.e., downstream of Harihara to Airani village. In this region we are receiving industrial effluent from Harihara poly fiber and grasim rayon industry along with domestic waste discharge into stream in Nalawagal, Nadiharalahalli and Airani and there is no other source of discharge and obstruction to stream water. We have selected six sampling stations based on maximum mixing of effluent along width and depth of stream water taken.

According to the outline in Equation (4), the BOD decay model outline was created by plotting $\ln L$ against (x/U) , and it resulted in a straight line with the slope of kr . [25]. For the purpose of this project, the computed average of the higher values of k in the initial stretch was calculated as the BOD loss rate. On the other hand, K_r and the computed average of lower values of k while journeying further downstream was calculated as the de-oxygenation constant K_d . DOBT was used to determine the reaeration constant ka , and the DO mass balance equation (Equation (5)) was used for the computations. The method requires the measurement of all the sources and sinks of DO. Only the reaeration values will not be measured. After that, the K_a is computed as the difference in the reaeration required to achieve the DO concentration that was measured at the downstream end of the stream segment under investigation. It is evident that this technique is equally effective when compared with other distributed equilibrium methods and tracer techniques. The light and dark bottle method was applied to establish the rates of respiration and photosynthesis. The results show that the river has a minimal amount of algae matter which could not lead to substantial variation in DO due to respiration and photosynthesis. After measuring all the variables were except K_a , using the least square was then applied to determine the estimated value of K_a for each data set.

Table 1. Some frequently of used predictive re-aeration equation

Investigator	Empirical formula
Connor and William Dobbins [14]	$Ka=3.901 U^{0.5} H^{-1.5}$
Churchill.et.al. [9]	$Ka=5.010 U^{0.969} H^{-1.673}$
Orlob & Krenkel et al. [15]	$Ka=173 (SU)^{0.404} H^{-0.66}$
Owens.et al. [16]	$Ka=5.35 U^{0.67} H^{-1.85}$
Langbein and Durum [17]	$Ka=5.14 U H^{-1.33}$
Cadwallader and McDonell [18]	$Ka=186 (SU)^{0.5} H^{-1.0}$
Edward L. Thackston [19]	$Ka=24.9 (1 + Fr^{0.5}) UH^{-1.0}$
J.D.Parkhurst & R. D. Pomeroy et al. [20]	$Ka=3 (1 + 0.17 Fr^2)(SU)^{0.375} H^{-1.0}$
Tassioglou & Wallace et al. [13]	$Ka=31,200 \text{ SU for } Q < 0.28 \text{ m}^3/\text{s}$ $Ka=15200 \text{ SU for } Q > .28 \text{ m}^3/\text{s}$
Smoot [21]	$Ka=543 S^{0.6236} U^{0.5325} H^{-0.7258}$
Moog [22]	$Ka=1740 U^{0.46} S^{0.79} H^{0.74}$ for $S < 0.00$
Jha et al. [23]	$Ka=5.791 U^{0.50} H^{-0.25}$
Jha et al. [24]	$Ka=0.603286 U^{0.4} S^{-1} H^{0.154}$ for $Fr < 1$

Where U=velocity in meter per sec, H=depth of waterway, S=stream bed slope, Fr=Froude number

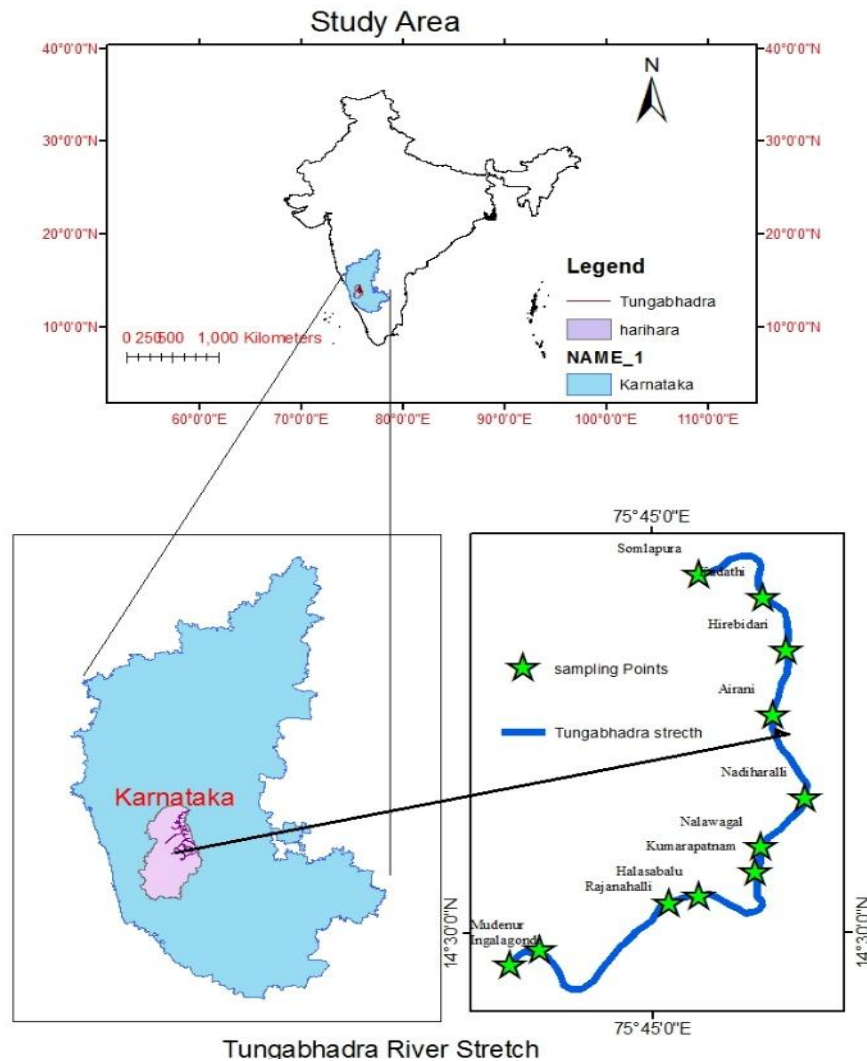


Fig. 1. location of river Tungabhadra selected for the study

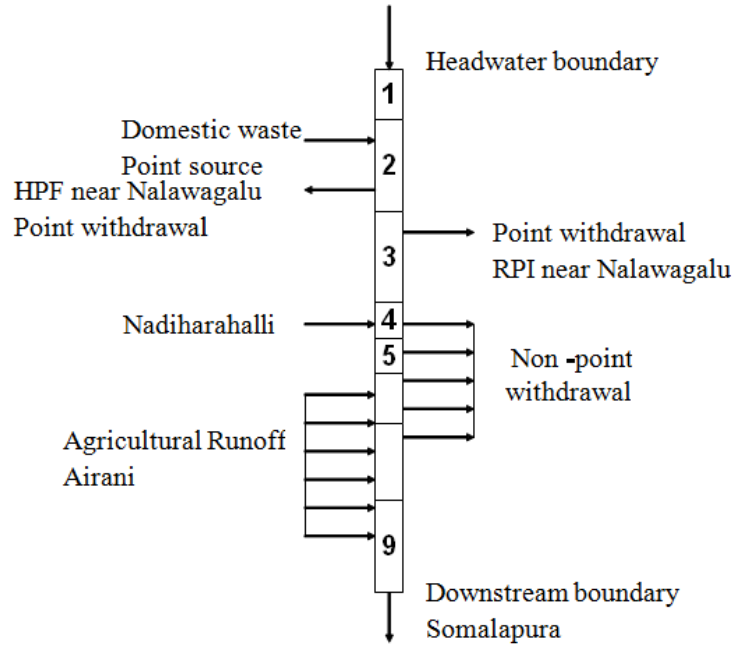


Fig. 2. Segmentation scheme to calculate K_a River

Table 2. K_a values measured by DOBT for different flow condition of river Tungabhadra

Sl. No	Velocity m/s	Depth in m	BED slope	Flow m ³ /s	K_a
January	0.51	0.22	0.0024	0.71	6.35
February	0.59	0.35	0.0024	0.64	6.1
March	0.62	0.28	0.0024	0.56	6.577
April	0.71	0.37	0.0024	0.548	6.6
may	0.73	0.5	0.0024	0.49	5.9
June	0.55	0.55	0.0024	0.497	6.57
July	0.56	0.6	0.0024	0.52	6.0794
October	0.57	0.61	0.0024	0.292	6.167
November	0.59	0.64	0.0024	0.46	4.1
December	0.6	0.66	0.0024	0.52	5.9

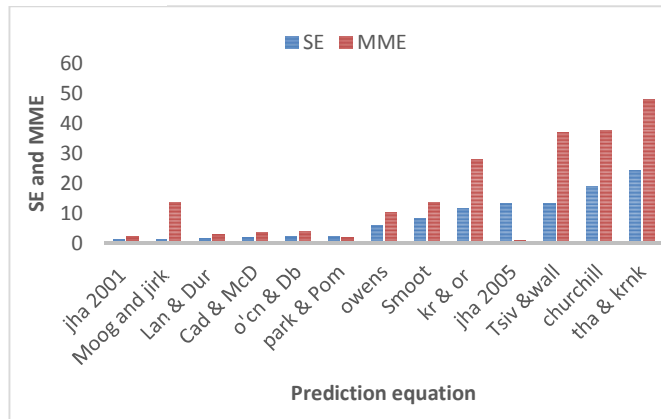


Fig. 3. Standard error and mean multiplicative error for predictive equation

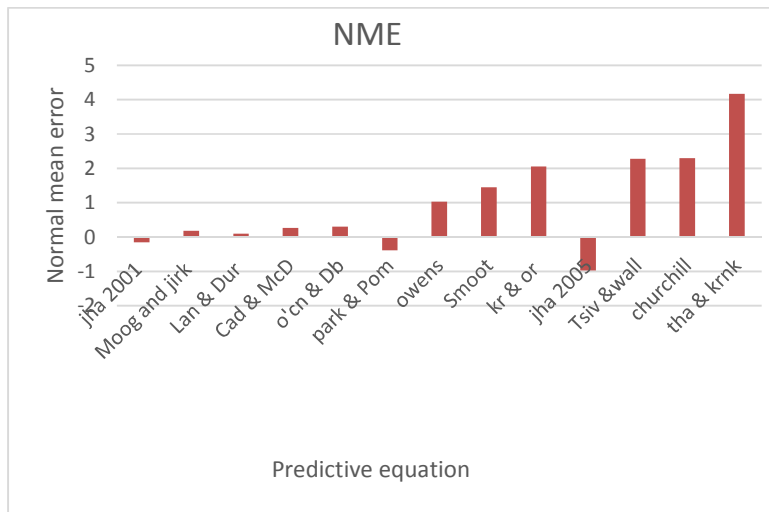


Fig. 4. Normal mean error for predictive equation

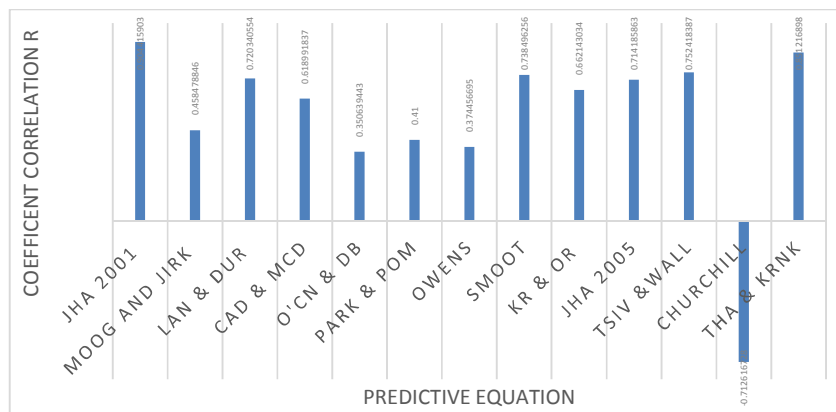


Fig. 5. Coefficient of correlation r value for predictive equation

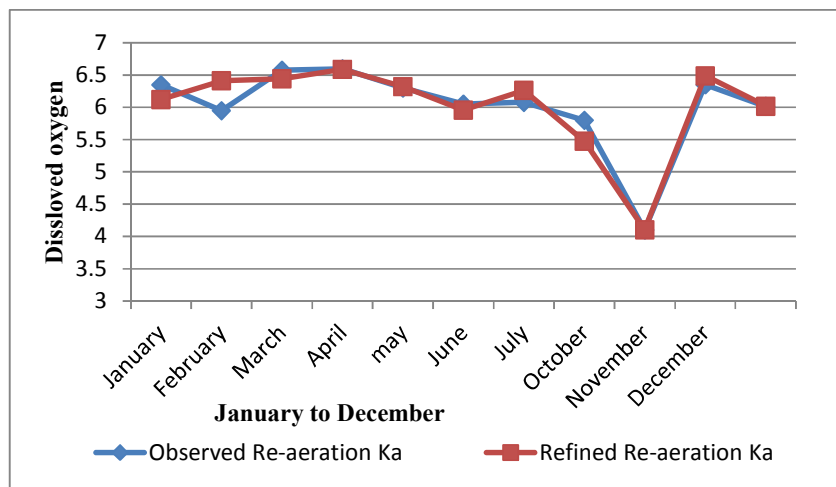


Fig. 6. Comparison of predicted and measured K_a Values

Evaluating and assessment error in predicted re-aeration equation

The performance of thirteen most popular equations are shown in Table 1 which have been evaluated by using statistical error method by application standard error, normal mean error method, mean multiplicative error method and correlation coefficient (r). The backdrop of this method is related to Jha [2001]. The Standard error and Normal mean error method are differential error which are calculated by using following equations.

$$SE = \sqrt{\sum_{i=1}^N ((Kp - Km)^2) / N} \quad (6)$$

$$NME = (100\% / 100 \sum_{i=1}^N ((Kp - Km)^2) / Km) \quad (7)$$

where N= number of reaerations, K_p= predicted value, K_m=measured value.

The mean multiplicative error method used to analysis result for estimation of impact inaccuracy [Moorg and Jirka].

$$MME = \{ \sum_{i=1}^N (\ln(Kp - Km)^2) / N \} \quad (8)$$

Correlation of coefficient (r) can be computed by pearson method. In this method we can check fractional variance (r) which must closer to unity which shows better result.

$$R = (1 - S_{ee} / S_{yy})^{0.5} \quad (9)$$

where S_{ee}=sum of square of difference between the observed and computed values and S_{yy}=sum of squares of departures of observed values of K_a from the mean of observed values.

ANALYSIS OF THE RESULT

Utilizing the measured information from the field survey i.e., stream velocity, flow, water depth, slope and along with physical-chemical and biological water quality parameter, information index from river Tungabhadra, reaeration rate coefficient (Ka) value were calculated for all thirteen reaeration equation which shown in Table 1.

Fig. 3 represents the graph of Mean Multiplicative Errors and Standard Errors (MME & SE) for Equations (6) and Equation (8). The chart in Fig. 4 represents the MME computed for Equation (7). On the other hand, the chart represented by Fig. 5 shows represents the

correlation coefficient r for the whole predictive equations.

It has been seen that the reaeration coefficient generated by Jha et al. [23] demonstrates the best concurrence with estimated esteems as far as SE, NME and MME (SE=1.24, NME=-0.159, MME=2.34) trailed by the condition created by Moog and jirk [22] (SE = 1.49, MME = 0.18, NME = 13.8). Notwithstanding, as far as correlation coefficient, the condition created by Jha et al. [23] demonstrates better assertion (r = 0.90) than that for the condition of Moog and jirk [22] (r = 0.45).

The Standard Error, Multiplicative mean error and Normal Mean error values obtained by the conditions proposed by Langbein and Durum's [17] (SE = 1.56, MME = 3.156 and NME = -0.096) are likewise nearer to the estimations of Jha et al. [23] and Cadwallader and McDonnell, [18]] (SE = 1.81, MME = 3.74, NME = 0.264). For O'Connor and Dobbins [7] condition (SE = 2.53, MME = 4.0519 and NME = 0.031), Parkhurst and Pomeroy, [20] yet the correlation coefficient is low (r = 0.4). The blunder value for the conditions proposed by different agents, specifically Churchill et al. [9] Tsvoglou and Wallace, [13] Krenkel and Orlob, [15] Owens et al. [16] Smoot, [21] and Thackston and Krenkel [19] are in fractional concurrence with the watched qualities.

The Standard Error, Multiplicative mean error and Normal Mean error values obtained by the conditions proposed by Langbein and Durum's [17] (SE = 1.56, MME = 3.156 and NME = -0.096) are likewise nearer to the estimations of Jha et al. [23] and Cadwallader and McDonnell, [18] (SE = 1.81, MME = 3.74, NME = 0.264). For O'Connor and Dobbins [7] condition (SE = 2.53, MME = 4.0519 and NME = 0.031), Parkhurst and Pomeroy, [20] yet the correlation coefficient is low (r = 0.4). The estimated errors for the conditions proposed by different authors, specifically Churchill et al. [9] Tsvoglou and Wallace, [13] Krenkel and Orlob, [15] Owens et al. [16] Smoot, [21] and Thackston and Krenkel [19] are in fractional concurrence with the observed qualities.

As the Standard Error and Normal Mean error values give differential-error [23], the outcomes are viewed as one-sided for bigger qualities and other side file down mistakes. The MME value, which utilizes the proportion of anticipated and estimated values, is viewed as most exact

criteria for error valuation [22]. If the MME is near unity, the model under use is said to deliver great outcomes. In the present work, the MME estimation of 2.34 for the condition proposed by Jha et al. [23] is nearest to the solidarity pursued by the estimations of 13.3, 1.9 (SEM and NME are in appropriate) and 2.47 for the conditions proposed by Moog and Jirka, [22] Parkhurst and Pomeroy [20] and Langbein and Durum, [17] individually. For the conditions proposed by O'Connor and Dobbins [14] the MME values are 4.0, separately. For the various outstanding conditions, the MME values are above 4.1. It might be seen that all the prescient conditions, with the exception of the condition proposed by Jha et al. [23] were created for the stream outside India. Since the present work is likewise embraced on a run of the mill Indian waterway, it is normal that the estimations of the present work are in nearest concurrence with the prescient condition proposed by Jha et al. [23] as the instrument of reaeration is same for the ordinary Indian topographical and climatic conditions.

After in-depth research that involves extensive field survey in a predefined segment of River Tungabhadra, an enhanced version of the predictive reaeration equation was developed. This new equation uses the flow depth and the velocity parameters only. Other parameters such as friction velocity, slope, and Froude number, were removed from the equation because they are related to the velocity and flow depth parameters. The updated version of the equation derived from the least-square technique is given in the chapters below.

$$K_a = 5.7126 U^{0.67691} H^{-0.69889} \quad (9)$$

The error estimates computed from equation (9) with the measured values in the present work are found to be improved relative to the literature equations given in Table 1. The qualities are Standard error= 0.407; Multiplicative mean error=0.98; Normal mean error=-0.019 and correlation coefficient $r = 0.91$. The examination of anticipated k_a esteemed utilizing the refined prescient Equation (9) and watched k_a esteems from the field information for 10 agent informational indexes is appeared in Fig. 6. The outcomes got are empowering and feature the better execution of the refined prescient condition.

3. CONCLUSION

The results derived from the popular predictive equation for reaeration constants found in the

literature were analysed using error estimation statistical methods. The results of the error estimations proved that the equation proposed by Jha et al. [23] has the highest correlation with the values measured for k_a . On the other hand, equations proposed Jha et al. [24] that make use of the slope parameters resulted in values that are significantly higher than the measured values. Other equations analysed with the statistical error estimation method are those proposed by Langbein and Durum [17], Moog and Jirka [22], and Parkhurst and Pomeroy, [20]. And they all showed high correlation with the measured values.

It is essential to note that the results derived from the enhanced reaeration equation that was designed for river Tungabhadra has higher levels of accuracy. It is recommended for use for future extensive field research in the river. The enhanced predictive reaeration equation that was created for River Tungabhadra is also valuable for other streams that have similar climatic, geographical, and hydraulic conditions.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Streeter HW, Phelps EB. A study of the pollution and natural purification of the Ohio River. Public Health Bulletin No.146. Washington (DC): Public Health Service; 1925
2. Jain SK, Jha R. Comparing the stream reaeration coefficient estimated from ANN and empirical models. Hydrol Sci. 2005; 50(6):1037–1052.
3. Connor ODJ, et al. Biological waste treatment. Tarry-town, NY: Pergamon; 1961.
4. Cleasby JL, et al. Oxygenation efficiency of bladed rotor. J Water Pollution Control Fed. 1968;40(3):412–424.
5. Chapra SC. Surface water quality modeling Singapore: McGraw-Hill International Editions; 1997
6. Danckwerts PV, et al. Significance of liquid film coefficients in gas absorption. Ind Eng Chem. 1951;43(6):1460–1467.
7. O'Connor DJ, Dobbins WE. Mechanism of reaeration in natural streams. Am Soc Civil Eng Trans. 1956;86(SA3):35–55
8. Streeter HW, Phelps EB. A study of the pollution and natural purification of the

- Ohio River. Public Health Bulletin No.146. Washington (DC): Public Health Service; 1925.
9. Churchill MA, Elmore HL, Buckingham RA. The prediction of stream reaeration rates. J Sanitary Eng Division. 1962;88(4):1–46.
 10. Issacs WP, Gaudy AF. Atmospheric oxygenation in a simulated stream. J Sanitary Eng Div. 1968;94(2):319–344.
 11. Edwards RW, Owens M, Gibbs JW. Estimates of surface aeration in two streams. J Inst Water Eng. 1961;15(5): 395–405.
 12. Zogorski JS, Faust SD. Atmospheric reaeration capacity of streams, Part II, Direct measurement of the atmospheric reaeration rate constant in the upper Raritan river basin. Environ Lett. 1973; 4(1):61–85.
 13. Tsivoglou EC, Wallace JR. Characterization of stream reaeration capacity. Report No. EPA-R3-72-012. Washington (DC): US Environmental Protection Agency; 1972
 14. Willam, Connor. Mechanism of reaeration in natural streams. Am Soc Civil Engineer Trans. 1958;123:641–684.
 15. Krenkel PA, et al. Turbulent diffusion and reaeration coefficient. J Sanitary Eng Div. 1962;88(2):53–83.
 16. Edwards RW, Gibbs JW. Some reaeration studies in streams. Internation Journal Air Water Pollution. 1964;8(8/9):469–486.
 17. Langbein WB, Durum WH. The American capacity of streams, USGS Circular No. 542. Washington (DC): United States Geological Survey; 1967.
 18. Cadwallader TE, McDonell AJ. A multivariate analysis of reaeration data. Water Res. 1969;3:731–742.
 19. Thackston EI, Krenkel PA. Reaeration prediction in natural streams. J Sanitary Eng Div. 1969;95(1):65–93.
 20. Parkhurst JD, Pomeroy RD. Oxygen absorption in streams. J Sanitary Eng Div. 1972;98(1):101–124F.
 21. Smoot H. An estimation of stream reaeration coefficient and hydraulic conditions in a pool and riffle stream Blacksburg (VA): Virginia Polytechnic Institute and State University; 1998.
 22. Moog DB, Jirka DH. Analysis of reaeration equations using mean multiplicative error. J Environ Eng Div. 1998;112(2):104–110.
 23. Jha R, Ojha CSP, Bhatia KKS. Refinement of predictive reaeration equations for a typical Indian river. Hydrol Process. 2001; 15(6):1047–1060.
 24. Jha R, Ojha. An supplementary approach for estimating reaeration rate coefficients. Hydrological Process. 2003;18:65–79
 25. Texas Water Development Board. Simulation of water quality in streams and canals. Report No. 128. Austin: Texas Department of Water Resources; 1971.APHA. "Standard methods for the examination of water and wastewater." 20th ed. Washington (DC): American Public Health Association; 1998
 26. APHA. Standard methods for the examination of water and wastewater. 20th ed. Washington (DC): American Public Health Association; 1998.

© 2019 Ranjith et al.; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Peer-review history:

The peer review history for this paper can be accessed here:
<http://www.sdiarticle3.com/review-history/49251>