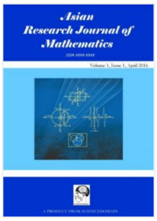

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# Effect of Variable Axial Force on the Deflection of Thick Beam Under Distributed Moving Load

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***Authors' contributions***

*This work was carried out in collaboration between all authors. Author JSA designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors JSA and STO managed the analysis of the study. Author AOO managed the literature searches. All authors read and approved the final manuscript.*

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## Abstract

The effect of variable axial force on the deflection of thick beam under moving load is investigated in this research work. In order to obtain solution to the dynamic problem, a technique base on the method of Galerkin with the series representation of Heaviside function, was first used to transform the equation and thereafter the transformed equations were solved using Struble's asymptotic method and Laplace transformation techniques in conjunction with convolution theory. The closed form solution of the transverse displacement for moving force and moving distributed mass models for the dynamical problem obtained were calculated for various time t. Important features of the analysis were investigated and discussed.

*Keywords: Axial force; moving load; struble's asymptotic method; convolution theory; Galerkins method.*

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# Nomenclatures

| | |
|---|---|
| $E(N/m^2)$ | Modulus of elasticity |
| $I(m^4)$ | Moment of inertia |
| $V(x,t)(m)$ | Beam Transverse displacement |
| $W(x,t)(m)$ | Beam Transverse rotation |
| $N_0(N)$ | Axial force |
| $K(N/m)$ | Foundation Stiffness |
| $G(N/m)$ | Shear modulus |
| $q(x,t)(Kg)$ | Traversing load |
| $\mu(Kg/m)$ | Mass per unit length |
| $g(m/s^2)$ | Acceleration due to gravity |
| $M(Kg)$ | Mass of the moving load |
| $H(x-ct)$ | Heaviside function |
| $x(m)$ | Spatial coordinate |
| $c(m/s)$ | Velocity |
| $t(s)$ | Time |
| $L(m)$ | Beam span |
| $\rho(Kg/m^3)$ | Density of the beam |
| $K^*$ | Constant dependent on the shape of the beam |
| $A(m^2)$ | Cross sectional area of the beam |
| $U_m(m)$ | mode function of the beam due to displacement |
| $Q_m(m)$ | mode function of the beam due to rotation |
| $\lambda_m$ | Mode frequency |
| $\delta(x-ct)$ | Dirac delta function |
| $\Gamma_0$ | Mass ratio of the structure-load system for the elastic beam |
| $\alpha_{mf1}$ | Natural frequency of the beam due to displacement |
| $\alpha_{mf2}$ | Natural frequency of the beam due to rotation |
| $\alpha_{mm}$ | Modified natural frequency of the beam due to moving mass |
| $\omega_{mf}$ | Modified frequency of the beam due to moving force |
| $\omega_{mm}$ | Modified frequency of the beam due to moving mass |

# 1 Introduction

Vibrations of beams are of considerable interest to the engineers designing mechanical and structural systems. Many researchers have investigated the free vibration analysis of beams having various boundary conditions and based on the Bernoulli-Euler beam theory (e.g [1-4] ). The well-known Bernoulli-Euler beam theory states that plane sections remain plane after deformation, regarding transverse shear strain to be neglected. Although this theory is very useful for slender beams and columns, it does not give accurate solutions for thick beams. In the Timoshenko beam theory, the normality assumption of the Bernoulli-Euler theory is relaxed and a constant state of transverse shear strain with respect to the thickness coordinate is included. The Timoshenko beam theory requires shear correction factors to compensate for the error due to this constant shear stress assumption. It should be noted that the bridges on which vehicles or trains travel and trolleys of overhead traveling cranes that move on their girders may be modeled as moving loads on simply supported beams. Comprehensive treatment of the subject of vibrations of structures due to moving loads which contains a large number of related cases is studied by Fryba [1]. A dynamic Green function approach is used to determine the response of a finite length of simply supported EulerBernoulli beam subjected to a moving load in [5] and the authors proposed a simple matrix expression for the deflection of the beam. In [6], the effect of a moving mass on the dynamic behavior of a simply supported Euler-Bernoulli beam was studied. The linear finite element analysis was used to investigate dynamic behavior of a simply supported Euler-Bernoulli beam under the act of moving loads by [7-10]. The linear vibration analysis using the Galerkins technique for a Timoshenko beam traversed by a moving mass was applied in [11] and this work was verified by [12] using the linear finite element method. Often, Engineers create artificial stresses in structures before loading, so that the stresses which then exist in the structures under load are more favorable than would otherwise be the case. Such artificial stresses are forces which may act axially or otherwise. When they act axially, they are called axial force. The artificial stresses are also called pre-stress. The aim of pre-stress structure is to limit tensile stresses and hence flexural cracking or bending in the structure under working conditions. Emphatically speaking the purpose of pre-stress is to external deformation and hence bending deformation.

If the beam is subjected to a force parallel to its axis in addition to the lateral loading. The local equilibrium of forces is altered because the axial force interacts with the lateral displacement to reduce an additional term . If N(x) is the variable axial force and V(x,t) is the axial displacement, the additional term is $\frac{\partial}{\partial x}\left[N(x)\frac{\partial V(x,t)}{\partial x}\right]$ if the pre-stress varies with partial coordinates and also becomes $N\frac{\partial^2 V(x,t)}{\partial x^2}$ if the pre-stress is a constant. Thus, in this paper we consider more realistic case where the axial force varies along the span of the beam. However, it is remarked at this juncture that in most of the existing literature in dynamics of structure under moving loads, moving loads have been idealized as moving concentrated loads which acts at a certain point on the structure and along the a single line in space . That is, the moving load is modeled as a lumped load. In practice, it is known that loads are actually distributed over a small segment or over the entire length of the structural member as they traverse the structure. Such moving loads are termed uniformly distributed loads. Concentrated forces are mere mathematical idealization, but cannot be found in the real world, where all forces are body forces acting over an area. Furthermore, in most of the investigations on the effect of axial force on a dynamic system, method of solution have been restricted to numerical simulating [13]. The loads moving on this elastic beam also be modeled realistically as a distributed load as against the unrealistic lump mass model that is common in literature.

Thus, this study is concerned with the variable axial force effect on the deflection of thick beam under moving load. The axial force is assumed to vary along this span of the beam. The three vital aspects of inertia terms are concerned. It is also assumed that the thick beam is of uniform cross-section and contain negligible dampy.

# 2 Mathematical Method

We consider the problem of axial force influence on response to distributed moving loads of thick beam and carrying a mass $\mu$. The transverse displacement V(x,t) of a uniform elastic beam of length L under the actions of mass M traveling at a uniform velocity c as shown in fig (1a and 1b) is governed by equation of the form [14].

$$\frac{\partial}{\partial x}\left[K^*GA\left(-\frac{\partial V(x,t)}{\partial x}+W(x,t)\right)\right]+\mu\frac{\partial^2 V(x,t)}{\partial t^2}+KV(x,t)-\frac{\partial}{\partial x}\left[N(x)\frac{\partial V(x,t)}{\partial x}\right]=q(x,t) \tag{2.1}$$

and

$$\frac{\partial}{\partial x}\left[EI\frac{\partial W(x,t)}{\partial x}\right]+K^*GA\left[-\frac{\partial V(x,t)}{\partial x}-W(x,t)\right]-I\rho\frac{\partial^2 W(x,t)}{\partial t^2}=0 \tag{2.2}$$

The boundary conditions of the above problem is simply supported and are assumed to be

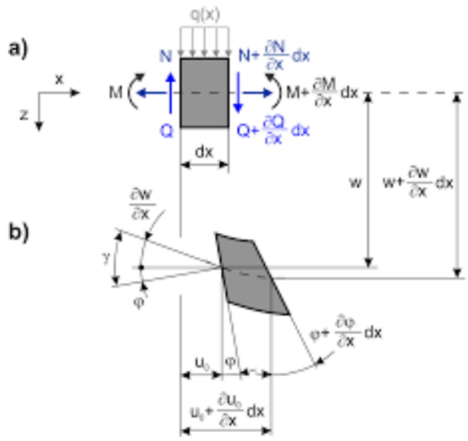

**Fig. 1. variable axial force on the deflection of thick beam**

$$V(0,t) = V(l,t) = 0 \ , \qquad \frac{\partial W(0,t)}{\partial x} = \frac{\partial W(l,t)}{\partial x} = 0 \tag{2.3}$$

The initial conditions however without any loss of generality is given by

$$V(x,0) = \dot{V}(x,0) = 0 \ , \qquad W(x,0) = \frac{\partial W(x,0)}{\partial t} = 0 \tag{2.4}$$

Since the inertia effects of the moving load is taken into consideration, therefore the load $q(x,t)$ takes the form [1]

$$q(x,t) = q_f(x,t)\left[1 - \frac{1}{g}\frac{d^2V(x,t)}{dt^2}\right] \tag{2.5}$$

the continuous moving force $q_f(x,t)$ acting on the beam model and the total derivatives in (2.1) and (2.5) are

$$q_f(x,t) = MgH\,(x - ct) \tag{2.6}$$

and

$$\frac{d^2V(x,t)}{dt^2} = \frac{\partial^2 V(x,t)}{\partial t^2} + 2c\frac{\partial^2 V(x,t)}{\partial x \partial t} + c^2\frac{\partial^2 V(x,t)}{\partial x^2} \tag{2.7}$$

where c is the velocity of the distributed mass, the time t is assumed to be limited to that interval of time within which the mass is on the beam, that is

$$0 \le ct \le L \tag{2.8}$$

g is the acceleration due to gravity, and $H(x - ct)$ is the Heaviside function defined as

$$H(x-ct) = \begin{cases} 0, & x < ct. \\ 1, & x \ge ct. \end{cases} \tag{2.9}$$

This describes the arrival of a continuous load on the beam, with the properties

$$\frac{d}{dx}H\left(x-ct\right)=\delta\left(x-ct\right) \tag{2.10}$$

$$f(x)H(x-ct)=\begin{cases}0, & x<ct.\\ 1, & x\geq ct.\end{cases} \tag{2.11}$$

Where $\delta(x-ct)$ represents the Dirac delta function and $H(x-ct)$ is a typical engineering function, called Heaviside function made to measure engineering application which often involve function that are either off or on. As an example in this problem, the pre-stress term is taken as [15]

$$N(x)=N_0\left(1+\sin\frac{\pi x}{l}\right) \tag{2.12}$$

where $N_0$ is the constant axial force.

Now using (2.5),(2.6),(2.7) and (2.8) in (2.1) and (2.2), we obtain

$$-K^*GA\frac{\partial^2 V(x,t)}{\partial x^2}+K^*GA\frac{\partial W(x,t)}{\partial x}+\mu\frac{\partial^2 V(x,t)}{\partial t^2}-\frac{\partial}{\partial x}\left[N_0\left(1+\sin\frac{\pi x}{l}\right)\frac{\partial V(x,t)}{\partial x}\right]$$
$$+KV(x,t)+MH\left[\frac{\partial^2 V(x,t)}{\partial t^2}+2c\frac{\partial^2 V(x,t)}{\partial x\partial t}+c^2\frac{\partial^2 V(x,t)}{\partial x^2}\right]=MgH(x,t) \tag{2.13}$$

and

$$EI\frac{\partial^2 W(x,t)}{\partial x^2}-K^*GA\frac{\partial V(x,t)}{\partial x}-K^*GAW\left(x,t\right)-I\rho\frac{\partial^2 W(x,t)}{\partial t^2}=0 \tag{2.14}$$

In (2.13) and (2.14), V(x,t) is the transverse displacement, t is the time, x is the spatial coordinate, c is the velocity, q(x,t) is the distributed load acting on the beam, W(x,t) is angular displacement, E is the Youngs modulus, I is the constant moment of inertia of the beam, $\mu$ is the constant mass per unit length of the beam, No is the constant axial force, K is the constant foundation stiffness, G is the constant shear modulus, $\rho$ is the density of the beam, M is the mass of the distributed load, g is the acceleration due to gravity, A is the cross sectional area of the beam, $K^*$ is the constant dependent on the shape of the cross-section, L is the length of the beam, t is the time taken, x is the spatial coordinate and H(x-ct) is the Heaviside function which describes the distributed load. N(x) Is the variable axial force.

## 2.1 Operational Simplification

Unlike cases where axial force is constant, finite integral transform is inapplicable and we resort to a modification of the approximate method best suited for solving diverse problem in dynamics of structures generally referred to as **Galerkin's Method**. Thus we use the Galerkin's method described in Oni and Awodola [16] to reduce the simultaneous partial differential equation to a sequence of second order ordinary simultaneous differential equation. Thus a solution of the form

$$V(x,t)=\sum_{m=1}^{n}Y_m(t)U_m(x) \tag{2.15}$$

and

$$W(x,t)=\sum_{m=1}^{n}Z_m(t)Q_m(x) \tag{2.16}$$

where $U_m(x)$ and $Q_m(x)$ are chosen such that the boundary conditions given are satisfied. For simply supported boundary conditions, due to displacement $U_m(x)$ is given as

$$U_m(x) = \sin\frac{m\pi x}{L} \tag{2.17}$$

and due to rotation $Q_m(x)$ is chosen as

$$Q_m(x) = \cos\frac{m\pi x}{L} \tag{2.18}$$

Equation (2.17) and (2.18) are chosen to satisfy the boundary conditions.Therefore by using (2.15), (2.16),(2.17) and (2.18) in (2.13) and (2.14) respectively,one obtains

$$\sum_{m=1}^{\infty}\Big[\mu U_m(x)\ddot{Y}_m(t) - K^*GA\,U_m^{''}(x)Y_m - N_0\Big(U_m^{''}(x) + \sin\frac{\pi x}{L}U_m^{''}(x) + \frac{\pi}{l}\cos\frac{\pi x}{L}U_m^{'}(x)\Big) + KV_mY_m(t) + K^*GA\,Q_m^{'}(x)Z_m(t) + M[H(x-ct)\,U_m(x)\ddot{Y}_m(t) + 2cH(x-ct)U_m^{'}(x)\dot{Y}_m(t) + c^2H(x-ct)U_m^{''}] = MgH(x-ct)\Big] \tag{2.19}$$

and

$$\sum_{m=1}^{\infty}\Big[\rho IQ_m(x)\ddot{Z}_m(t) + K^*GA\,Q_m(x)Z_m(t) - EIQ_m^{''}(x)Z_m(t) - K^*GAU_m^{'}(x)Y_m(t)\Big] = 0 \tag{2.20}$$

In order to determine an expression for $Y_m(t)$ and $Z_m(t)$, it is required that the expression on the left hand side of equations (2.19) and (2.20) are orthogonal to the function $\big(U_k(x) = \sin\frac{k\pi x}{L}\big)$ and $\big(Q_k(x) = \cos\frac{k\pi x}{L}\big)$ respectively. Thus we write

$$\int_0^L\Bigg[\sum_{m=1}^{\infty}\Big[\mu U_m(x)\ddot{Y}_m(t) - K^*GA\,U_m^{''}(x)Y_m - N_0\Big(U_m^{''}(x) + \sin\frac{\pi x}{L}U_m^{''}(x) + \frac{\pi}{l}\cos\frac{\pi x}{L}U_m^{'}(x)\Big) + KU_mY_m(t) + M[H(x-ct)\,U_m(x)\ddot{Y}_m(t) + 2cH(x-ct)U_m^{'}(x)\dot{Y}_m(t) + c^2H(x-ct)U_m^{''}] = MgH(x-ct)\Big]\Bigg]U_k(x)dx - \int_0^L\Big[\{K^*GA\,Q_m^{'}(x)Z_m(t)\}\Big]Q_k(x)dx \tag{2.21}$$

and

$$\int_0^L\Bigg\{\sum_{m=1}^{\infty}\Big[\rho IQ_m(x)\ddot{Z}_m(t) + K^*GA\,Q_m(x)Z_m(t) - EIQ_m^{''}(x)Z_m(t)\Big]\Bigg\}Q_k(x)dx - \int_0^L\Bigg\{\sum_{m=1}^{\infty}\Big[K^*GAU_m^{'}(x)Y_m(t).U_k(x)dx\Big]\Bigg\} = 0 \tag{2.22}$$

In view of (2.17) to (2.18) and their derivatives, one obtain

$$\sum_{m=1}^{\infty}\Bigg\{H_0(m,k)\ddot{Y}_m(t) + H_1(m,k)Y_m(t) + \frac{M}{\mu}\Big[H_2(m,k)\ddot{Y}_m(t) + 2cH_3(m,k)\dot{Y}_m(t) + c^2H_4(m,k)Y_m(t)\Big] + H_6(m,k)Z_m(t)\Bigg\} = \frac{M}{\mu}gH_5(m,k) \tag{2.23}$$

and

$$\sum_{m=1}^{\infty}\left\{\Big[H_7(m,k)\ddot{Z}_m(t)+H_8(m,k)Z_m(t)\Big]-H_9(m,k)Y_m(t)\right\}=0 \tag{2.24}$$

where;

$$H_1(m,k)=H_{1A}(m,k)-H_{1B}(m,k)-H_{1C}(m,k)-H_{1D}(m,k)-H_{1E}(m,k) \tag{2.25}$$

$$H_8(m,k)=H_{8A}(m,k)-H_{8B}(m,k) \qquad H_0(m,k)=\int_0^L U_m(x)\,U_k(x)\,dx \tag{2.26}$$

$$H_{1A}(m,k)=\frac{K}{\mu}\int_0^L U_m(x)\,U_k(x)\,dx \qquad H_{1B}(m,k)=\frac{K^*GA}{\mu}\int_0^L U_m^{''}(x)\,U_k(x)\,dx \tag{2.27}$$

$$H_{1C}(m,k)=\frac{N_0}{\mu}\int_0^L U_m^{''}(x)\,U_k(x)\,dx \qquad H_{1D}(m,k)=\frac{N_0}{\mu}\int_0^L \sin\frac{\pi x}{L}U_m^{''}(x)\,U_k(x)\,dx \tag{2.28}$$

$$H_{1E}(m,k)=\frac{N_0\pi}{\mu L}\int_0^L \cos\frac{\pi x}{L}U_m^{'}(x)\,U_k(x)\,dx \qquad H_2(m,k)=\int_0^L H(x-ct)\,U_m(x)\,U_k(x)\,dx \tag{2.29}$$

$$H_3(m,k)=\int_0^L H(x-ct)\,U_m^{'}(x)\,U_k(x)\,dx \qquad H_4(m,k)=\int_0^L H(x-ct)\,U_m^{''}(x)\,U_k(x)\,dx \tag{2.30}$$

$$H_5(m,k)=\int_0^L H(x-ct)\,U_k(x)\,dx \qquad H_6(m,k)=\frac{K^*G\,A}{\mu}\int_0^L Q_m^{''}(x)\,Q_k(x)\,dx \tag{2.31}$$

$$H_7(m,k)=\int_0^L Q_m^{''}(x)\,Q_k(x)\,dx \qquad H_{8A}(m,k)=\frac{K^*G\,A}{\rho I}\int_0^L Q_m(x)\,Q_k(x)\,dx \tag{2.32}$$

$$H_{8B}(m,k)=\frac{E}{\rho}\int_0^L Q_m^{''}(x)\,Q_k(x)\,dx \qquad H_9(m,k)=\frac{K^*G\,A}{\rho I}\int_0^L U_m^{'}(x)\,U_k(x)\,dx \tag{2.33}$$

Using the property of Heaviside function, it can be expressed in series form given by [17] i.e

$$H(x-ct)=\frac{1}{4}+\frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi(x-ct)}{2n+1} \tag{2.34}$$

It can be shown that

$$H_2(m,k)=\psi_{1A}(m,k)+\frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\psi_{1B}(n,m,k)-\frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\psi_{1C}(n,m,k) \tag{2.35}$$

$$H_3(m,k)=\psi_{2A}(m,k)+\frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\psi_{2B}(n,m,k)-\frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\psi_{2C}(n,m,k) \tag{2.36}$$

$$H_4(m,k) = \psi_{3A}(m,k) + \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\psi_{3B}(n,m,k) - \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\psi_{3C}(n,m,k) \quad (2.37)$$

where;

$$\psi_{1A}(m,k) = \frac{1}{4}\int_o^L \sin\frac{m\pi x}{L}\sin\frac{k\pi x}{L}\,dx\,, \quad \psi_{1B}(n,m,k) = \int_o^L \sin(2n+1)\pi x\,\sin\frac{m\pi x}{L}\,\sin\frac{k\pi x}{L}\,dx \quad (2.38)$$

$$\psi_{1C}(n,m,k) = \int_o^L \cos(2n+1)\pi x\,\sin\frac{m\pi x}{L}\,\sin\frac{k\pi x}{L}\,dx, \quad \psi_{2A}(m,k) = \frac{m\pi}{4L}\int_o^L \cos\frac{m\pi x}{L}\sin\frac{k\pi x}{L}\,dx \quad (2.39)$$

$$\psi_{2B}(n,m,k) = \frac{m\pi}{L}\int_o^L \sin(2n+1)\pi x\,\cos\frac{m\pi x}{L}\sin\frac{k\pi x}{L}\,dx \quad (2.40)$$

$$\psi_{2C}(n,m,k) = \frac{m\pi}{L}\int_o^L \cos(2n+1)\pi x\,\cos\frac{m\pi x}{L}\sin\frac{k\pi x}{L}\,dx \quad (2.41)$$

$$\psi_{3A}(m,k) = -\frac{m^2\pi^2}{4L^2}\int_o^L \sin\frac{m\pi x}{L}\sin\frac{k\pi x}{L}\,dx \quad (2.42)$$

$$\psi_{3B}(n,m,k) = -\frac{m^2\pi^2}{L^2}\int_o^L \sin(2n+1)\pi x\,\sin\frac{m\pi x}{L}\sin\frac{k\pi x}{L}\,dx \quad (2.43)$$

$$\psi_{3C}(n,m,k) = -\frac{m^2\pi^2}{L^2}\int_o^L \cos(2n+1)\pi x\,\sin\frac{m\pi x}{L}\sin\frac{k\pi x}{L}\,dx \quad (2.44)$$

Further simplification and rearrangement of equations (2.23) and (2.24) using the solutions of the integrals listed above, it gives

$$\sum_{m=1}^{\infty}\Bigg\{H_0'(m,k)\ddot{Y}_m(t) + H_1'(m,k)Y_m(t) + \Gamma_0\Bigg[\Big[\psi_{1A}'(m,k) + \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\psi_{1B}'(n,m,k)$$
$$-\frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\psi_{1C}'(n,m,k)\Big]\ddot{Y}_m(t) + 2c\Big[\psi_{2A}'(m,k) + \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\psi_{2B}'(n,m,k) \quad (2.45)$$
$$-\frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\psi_{2C}'(n,m,k)\Big]\dot{Y}_m(t) + c^2\Big[\psi_{3A}'(m,k) + \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\psi_{3B}'(n,m,k)$$
$$-\frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\psi_{3C}'(n,m,k)\Big]Y_m(t)\Bigg] + H_6'(m,k)Z_m(t) = \frac{MgL}{\mu\pi k}\Big[\cos m\pi + \cos\frac{k\pi ct}{L}\Big]\Bigg\}$$

and

$$\sum_{m=1}^{\infty}\Bigg\{\Big[H_7'(m,k)\ddot{Z}_(m)(t) + H_8'Z_m(t)\Big] - H_9'(m,k)Y_m(t)\Bigg\} = 0 \quad (2.46)$$

where;

$$\Gamma_0 = \frac{M}{\mu L} \quad (2.47)$$

Equations (2.45) and (2.46) are now the fundamental equations governing the dynamic problem of the Axial force on the response to moving distributed load of Timoshenko beam type resting on Vlasov foundation. Equation (61) coupled non-homogeneous second order ordinary differential equation hold for the classical boundary condition. It follows that two special cases of the equation (2.45) arise, namely, the **moving force** problem (i.e. when all the inertia terms are neglected) and **moving mass** problem (i.e. when all the inertia terms are included).

## 2.2 Solution of the Transformed Equation

### 2.2.1 Thick beam traversed by moving distributed force

In this section, an approximate model of the differential equation describing the response of a uniform Timoshenko beam resting on Vlasov foundation and under the action of a moving distributed load will be obtained by neglecting inertia effect $\Gamma_0 = 0$, then equations (2.45) and (2.46) reduces to

$$\sum_{m=1}^{\infty}\left\{H_0^{'}(m,k)\ddot{Y}_m(t) + H_1^{'}(m,k)Y_m(t) + H_6^{'}(m,k)Z_m(t) = \frac{MgL}{\mu\pi k}\left[\cos m\pi + \cos\frac{k\pi ct}{L}\right]\right\} \quad (2.48)$$

and

$$\sum_{m=1}^{\infty}\left\{\left[H_7^{'}(m,k)\ddot{Z}_(m)(t) + H_8^{'}Z_m(t)\right] - H_9^{'}(m,k)Y_m(t)\right\} = 0 \quad (2.49)$$

This is the classical case of a moving force problem associated with the system. It is an approximated model which assume the inertia effect of the moving mass is negligible. In order to solve equations (2.48) and (2.49), the method of Laplace transform is resorted to rearranging the two equations, one obtains By rearranging the two equations, one will obtain

$$\ddot{Y}_m(t) + \alpha_{mf1}^2 Y_m(t) + \alpha_a Z_m(t) = \frac{MgL}{\mu\pi k H_0^{'}(m,k)}\left(R_m + \cos\theta_k(t)\right) \quad (2.50)$$

and

$$\ddot{Z}_m(t) + \alpha_{mf2}^2 Z_m(t) - \alpha_b Y_m(t) = 0 \quad (2.51)$$

where;

$$\alpha_{mf1}^2 = \frac{H_1^{'}(m,k)}{H_0^{'}(m,k)}\ ; \qquad \alpha_{mf2}^2 = \frac{H_8^{'}(m,k)}{H_7^{'}(m,k)} \quad (2.52)$$

$$\alpha_a = \frac{H_6^{'}(m,k)}{H_0^{'}(m,k)}\ ; \qquad \alpha_b = \frac{H_9^{'}(m,k)}{H_7^{'}(m,k)} \quad (2.53)$$

$$R_m = -(-1)^m\ ; \qquad \theta_k = \frac{k\pi c}{L} \quad (2.54)$$

Equations (2.50) and (2.51) are second order ordinary differential equations, therefore, subjecting the two equations to a Laplace transform defined as

$$\Lambda = \int_0^{\infty} (.)\, e^{-st} dt \quad (2.55)$$

where s is the Laplace parameter. By applying the initial conditions (2.4), one will obtain an algebraic equations given by

$$\bar{Y}_m(s)\left(s^2 + \alpha_{mf1}^2\right) + \alpha_a \bar{Z}_m(s) = \frac{MgL}{\mu\pi k H_0^{'}(m,k)}\left(\frac{R_m}{s} + \frac{s}{s^2 + \theta_k^2}\right) \quad (2.56)$$

and

$$\bar{Z}_m(s)\left(s^2+\alpha_{mf2}^2\right)-\alpha_b\bar{Y}_m(s)=0 \tag{2.57}$$

Solving equations (2.56) and (2.57) simultaneously by eliminating parameter $\bar{Y}_m(s)$, one obtains

$$\bar{Z}_m(s)\left(S^2+\omega_{mf}^2\right)^2=\alpha_{m2}\,q_f\left(\frac{R_m}{S}+\frac{s}{S^2+\theta_k^2}\right) \tag{2.58}$$

where;

$$\omega_{mf1}^2=\frac{1}{2}\left[-\left(\alpha_{mf1}^2+\alpha_{mf2}^2\right)+\sqrt{\left(\alpha_{mf1}^2-\alpha_{mf2}^2\right)^2-4\alpha_a\alpha_b}\right] \tag{2.59}$$

$$\omega_{mf2}^2=-\frac{1}{2}\left[\left(\alpha_{mf1}^2+\alpha_{mf2}^2\right)+\sqrt{\left(\alpha_{mf1}^2-\alpha_{mf2}^2\right)^2-4\alpha_a\alpha_b}\right] \tag{2.60}$$

and

$$Q_f=\frac{MgL}{\mu\pi kH_0'(m,k)} \tag{2.61}$$

Further rearrangement of (2.58), yields

$$\bar{Z}_m(s)=\frac{\alpha_b\,Q_f}{\omega_{mf1}^2-\omega_{mf2}^2}\left[Q_{1A}+Q_{1B}\right] \tag{2.62}$$

where;

$$Q_{1A}=\frac{R_m}{S}\left[\frac{1}{\left(S^2+\omega_{mf2}^2\right)}-\frac{1}{\left(S^2+\omega_{mf1}^2\right)}\right];\qquad Q_{1B}=\frac{S}{S^2+\theta_k^2}\left[\frac{1}{\left(S^2+\omega_{mf2}^2\right)}-\frac{1}{\left(S^2+\omega_{mf1}^2\right)}\right] \tag{2.63}$$

Thus, the equation reduces to that of finding the Laplace inversion of (2.62). So that the Laplace inversion of $Z_m(s)$ is the convolution of f(s) and g(s) defined as;

$$f(s)*g(s)=\int_0^L f(t-u)\,g(u)\,du \tag{2.64}$$

using (2.64), $Z_m(s)$ is easily expressed as;

$$Z_m(t)=\frac{\alpha_b\,Q_f}{\omega_{mf1}^2-\omega_{mf2}^2}\left[Q_{1a}+Q_{1b}\right] \tag{2.65}$$

where;

$$\begin{aligned}Q_{1a}=R_m\Bigg\{&\frac{1}{\omega_{mf2}}\left[\sin\omega_{mf2}t\int_0^t\cos\omega_{mf2}u\,du-\cos\omega_{mf2}t\int_0^t\sin\omega_{mf2}u\,du\right]\\&-\frac{1}{\omega_{mf1}}\left[\sin\omega_{mf1}t\int_0^t\cos\omega_{mf1}u\,du-\cos\omega_{mf1}t\int_0^t\sin\omega_{mf1}u\,du\right]\Bigg\}\end{aligned} \tag{2.66}$$

$$\begin{aligned}Q_{1b}=\Bigg\{&\frac{1}{\omega_{mf2}}\left[\sin\omega_{mf2}t\int_0^t\cos\omega_{mf2}u\,\cos\theta_k\,du-\cos\omega_{mf2}t\int_0^t\sin\omega_{mf2}u\,\cos\theta_k\,du\right]\\&-\frac{1}{\omega_{mf1}}\left[\sin\omega_{mf1}t\int_0^t\cos\omega_{mf1}u\,\cos\theta_k\,du-\cos\omega_{mf1}t\int_0^t\sin\omega_{mf1}u\,\cos\theta_k\,du\right]\Bigg\}\end{aligned} \tag{2.67}$$

Solving the integrals in $Q_{1a}\, and\, Q_{1b}$, thereafter put the solution back into equation (2.65) and evaluate with some rearrangements, one obtains

$$Z_m(t) = \frac{\alpha_b Q_f}{\omega_{mf1}^2\omega_{mf2}^2(\omega_{mf1}^2-\omega_{mf2}^2)\omega_{1\theta}^2\omega_{2\theta}^2}\Bigg\{R_m\omega_{1\theta}^2\omega_{2\theta}^2\Big[\omega_{mf2}^2\cos\omega_{mf1}t-\omega_{mf1}^2\cos\omega_{mf2}t$$
$$+(\omega_{mf1}^2-\omega_{mf2}^2)\Big]+\omega_{mf1}^2\omega_{mf2}^2\Big[\omega_{1\theta}^2(\cos\theta_k t-\cos\omega_{mf2}t)-\omega_{2\theta}^2(\cos\theta_k t-\cos\omega_{mf1}t)\Big]\Bigg\} \tag{2.68}$$

where;

$$\omega_{1\theta}^2 = (\omega_{mf1}^2-\theta_k^2); \qquad \omega_{2\theta}^2 = (\omega_{mf2}^2-\theta_k^2) \tag{2.69}$$

Substituting (2.68) into (2.16), one obtains

$$W(x,t) = \sum_{m=1}^{\infty}\frac{\alpha_b Q_f}{\omega_{mf1}^2\omega_{mf2}^2(\omega_{mf1}^2-\omega_{mf2}^2)\omega_{1\theta}^2\omega_{2\theta}^2}\Bigg\{R_m\omega_{1\theta}^2\omega_{2\theta}^2\Big[\omega_{mf2}^2\cos\omega_{mf1}t-\omega_{mf1}^2\cos\omega_{mf2}t$$
$$+(\omega_{mf1}^2-\omega_{mf2}^2)\Big]+\omega_{mf1}^2\omega_{mf2}^2\Big[\omega_{1\theta}^2(\cos\theta_k t-\cos\omega_{mf2}t)-\omega_{2\theta}^2(\cos\theta_k t-\cos\omega_{mf1}t)\Big]\Bigg\} * \cos\frac{m\pi x}{L} \tag{2.70}$$

Equation (2.70) represent the angular displacement of the thick beam under the action of moving distributed force for the dynamic system.

In order to get the transverse displacement of the dynamic problem, we eliminate $\bar{Z}_m(s)$ in simultaneous equations (2.56) and (2.57), then after some simplification and rearrangement, one obtains

$$\bar{Y}_m(s) = Q_f\Big[Q_{2A}+Q_{2B}\Big]+\alpha_{mf}^2\,Q_f\Big[Q_{1A}+Q_{1B}\Big] \tag{2.71}$$

where;

$$Q_{1A} = \frac{R_m}{S}\left[\frac{1}{(S^2+\omega_{mf2}^2)}-\frac{1}{(S^2+\omega_{mf1}^2)}\right], \qquad Q_{1B} = \frac{S}{S^2+\theta_k^2}\left[\frac{1}{(S^2+\omega_{mf2}^2)}-\frac{1}{(S^2+\omega_{mf1}^2)}\right] \tag{2.72}$$

$$Q_{1A} = \frac{R_m}{S}\left[\frac{\omega_{mf1}^2}{(S^2+\omega_{mf1}^2)}-\frac{\omega_{mf2}^2}{(S^2+\omega_{mf2}^2)}\right] \qquad Q_{1B} = \frac{S}{S^2+\theta_k^2}\left[\frac{\omega_{mf1}^2}{(S^2+\omega_{mf1}^2)}-\frac{\omega_{mf2}^2}{(S^2+\omega_{mf2}^2)}\right] \tag{2.73}$$

Using (2.64), on (2.73), then $\bar{Y}_m(s)$ is easily expressed as

$$\bar{Y}_m(s) = Q_f\Big[Q_{2a}+Q_{2b}\Big]+\alpha_{mf}^2\,Q_f\Big[Q_{1a}+Q_{1b}\Big] \tag{2.74}$$

Since the terms $Q_{1A}\, and Q_{1b}$ has been solved, then we are left with terms $Q_{2a}\, and Q_{2b}$ which are given as

$$Q_{2a} = R_m\Bigg\{\omega_{mf1}\Bigg[\sin\omega_{mf1}t\int_0^t\cos\omega_{mf1}u\,du-\cos\omega_{mf1}t\int_0^t\sin\omega_{mf1}u\,du\Bigg]$$
$$+\omega_{mf2}\Bigg[\sin\omega_{mf2}t\int_0^t\cos\omega_{mf2}u\,du+\cos\omega_{mf2}t\int_0^t\sin\omega_{mf2}u\,du\Bigg]\Bigg\} \tag{2.75}$$

and

$$Q_{2b} = \omega_{mf1}\Big[\sin\omega_{mf1}t\int_0^t \cos\omega_{mf1}u\cos\theta_k u\,du - \cos\omega_{mf1}t\int_0^t \sin\omega_{mf1}u\cos\theta_k u\,du\Big]$$
$$-\omega_{mf2}\Big[\sin\omega_{mf2}t\int_0^t \cos\omega_{mf2}u\cos\theta_k u\,du - \cos\omega_{mf2}t\int_0^t \sin\omega_{mf2}u\cos\theta_k u\,du\Big] \quad (2.76)$$

It is easily shown that

$$Q_{1a} = \frac{R_m}{\omega_{mf1}^2\omega_{mf2}^2}\Big[\omega_{mf2}^2\cos\omega_{mf1}t - \omega_{mf1}^2\cos\omega_{mf2}t + (\omega_{mf1}^2-\omega_{mf2}^2)\Big] \quad (2.77)$$

$$Q_{1b} = \frac{R_m}{(\omega_{mf1}^2-\theta_k^2)(\omega_{mf2}^2-\theta_k^2)}\Big[(\omega_{mf1}^2-\theta_k^2)(\cos\theta_k t - \cos\omega_{mf2}t)$$
$$-(\omega_{mf2}^2-\theta_k^2)(\cos\theta_k t - \cos\omega_{mf1}t)\Big] \quad (2.78)$$

$$Q_{2a} = R_m\Big(\cos\omega_{mf2}t - \cos\omega_{mf1}t\Big) \quad (2.79)$$

and

$$Q_{2b} = \frac{1}{\omega_{1\theta}^2\omega_{2\theta}^2}\Big[\omega_{mf1}^2\omega_{2\theta}^2(\cos\theta_k t - \cos\omega_{mf1}t) - \omega_{mf2}^2\omega_{1\theta}^2(\cos\theta_k t - \cos\omega_{mf2}t)\Big] \quad (2.80)$$

Substituting equations (2.77), (2.78), (2.79) and (2.80) into equation (2.74), after some simplification, one obtains

$$Y_m(t) = \frac{Q_f}{\omega_{mf1}^2\omega_{mf2}^2\omega_{mfd}^2\omega_{1\theta}^2\omega_{2\theta}^2}\Bigg\{R_m\omega_{1\theta}^2\omega_{2\theta}^2\Big[\omega_{mf2}^2\Big(\omega_{mf1}^2\cos\omega_{mf2}t - \alpha_{mf2}^2\cos\omega_{mf1}t\Big)$$
$$-\omega_{mf1}^2\Big(\omega_{mf2}^2\cos\omega_{mf1}t + \alpha_{mf2}^2\cos\omega_{mf2}t\Big) + \alpha_{mf2}^2\Big(\omega_{mf1}^2-\omega_{mf2}^2\Big)\Big] \quad (2.81)$$
$$+\omega_{mf1}^2\omega_{mf2}^2\Big(\omega_{mf1}^2-\alpha_{mf2}^2\Big)\Big[\omega_{2\theta}^2(\cos\theta_k t - \cos\omega_{mf1}t) - \omega_{1\theta}^2(\cos\theta_k t - \cos\omega_{mf2}t)\Big]\Bigg\}$$

where

$$\omega_{mfd}^2 = (\omega_{mf1}^2 - \omega_{mf2}^2) \quad (2.82)$$

Putting equation (2.82) into (2.15), one obtains

$$V(x,t) = \sum_{m=1}^{\infty}\frac{Q_f}{\omega_{mf1}^2\omega_{mf2}^2\omega_{mfd}^2\omega_{1\theta}^2\omega_{2\theta}^2}\Bigg\{R_m\omega_{1\theta}^2\omega_{2\theta}^2\Big[\omega_{mf2}^2\Big(\omega_{mf1}^2\cos\omega_{mf2}t - \alpha_{mf2}^2\cos\omega_{mf1}t\Big)$$
$$-\omega_{mf1}^2\Big(\omega_{mf2}^2\cos\omega_{mf1}t + \alpha_{mf2}^2\cos\omega_{mf2}t\Big) + \alpha_{mf2}^2\Big(\omega_{mf1}^2-\omega_{mf2}^2\Big)\Big]$$
$$+\omega_{mf1}^2\omega_{mf2}^2\Big(\omega_{mf1}^2-\alpha_{mf2}^2\Big)\Big[\omega_{2\theta}^2(\cos\theta_k t - \cos\omega_{mf1}t) - \omega_{1\theta}^2(\cos\theta_k t - \cos\omega_{mf2}t)\Big]\Bigg\} * \sin\frac{m\pi x}{L} \quad (2.83)$$

Equation (2.83) represent the transverse displacement of the thick beam under the action of moving distributed force for the dynamic system.

### 2.2.2 Thick beam traversed by moving distributed mass

In this section, the solution to the entire equation (2.45) is sought when no terms of the coupled differential equation is neglected. An approximate analytical solution to Eqn. (2.45) is resort to. Thus, we used a modification of the asymptotic method due to strubble's technique which is often used in treating oscillatory system. To this ends, equation (2.45) is rearranged to take the form

$$\ddot{Y}_m(t) + \frac{2\Gamma_0 c Q_2(n,m,k)}{H_0^{'}(m,k) + \Gamma_0 Q_1(n,m,k)}\dot{Y}_m(t) + \frac{H_1^{'}(m,k) + \Gamma_0 c^2 Q_3(n,m,k)}{H_0^{'}(m,k) + \Gamma_0 Q_1(n,m,k)}Y_m(t)$$
$$= \frac{MgL}{\mu\pi k\left(H_0{'}(m,k) + \Gamma_0 Q_1(n,m,k)\right)}\Big[R_m + \cos\theta_k t\Big] \quad (2.84)$$

where;

$$Q_1(m,k) = \psi_{1A}^{'}(m,k) + \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\psi_{1B}^{'}(n,m,k) - \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\psi_{1C}^{'}(n,m,k) \quad (2.85)$$

$$Q_2(m,k) = \psi_{2A}^{'}(m,k) + \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\psi_{2B}^{'}(n,m,k) - \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\psi_{2C}^{'}(n,m,k) \quad (2.86)$$

$$Q_3(m,k) = \psi_{3A}^{'}(m,k) + \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\psi_{3B}^{'}(n,m,k) - \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\psi_{3C}^{'}(n,m,k) \quad (2.87)$$

With this technique, one seeks the modified frequency corresponding to the frequency of the free system due to presence of the effect of axial force N(x).

An equivalent free system operator defined by the modified frequency, then replace equation (2.85). Thus the right hand side of (2.85) is set to zero for homogeneous equation and a parameter $\epsilon < 1$ is considered for any arbitrary mass ratio $\Gamma_0$ defined as [16];

$$\epsilon = \frac{\Gamma_0}{1+\Gamma_0} \quad (2.88)$$

it can be shown that

$$\Gamma_0 = \epsilon + 0(\epsilon^2) \quad (2.89)$$

and

$$\frac{1}{H_0^{'}(m,k) + \epsilon Q_1(n,m,k)} = \frac{1}{H_0^{'}(m,k)}\left[1 - \frac{\epsilon Q_1(n,m,k)}{H_0^{'}(m,k)}\right] \quad (2.90)$$

whenever

$$\left|1 - \frac{\epsilon Q_1(n,m,k)}{H_0^{'}(m,k)}\right| < 1 \quad (2.91)$$

Using equations (2.89) and (2.90) in (2.84), we have

$$\ddot{Y}_m(t) + \frac{1}{H_0^{'}(m,k)}\left[1 - \frac{\epsilon Q_1(n,m,k)}{H_0^{'}(m,k)}\right](2c\epsilon Q_2(n,m,k))\,\dot{Y_m}(t)$$
$$+\frac{1}{H_0^{'}(m,k)}\left[1 - \frac{\epsilon Q_1(n,m,k)}{H_0^{'}(m,k)}\right]\Big(H_1^{'}(m,k) + \epsilon c^2 Q_3(n,m,k)\Big)\,Y_m(t)$$
$$= \frac{MgL}{\mu\pi k}\frac{1}{H_0^{'}(m,k)}\left[1 - \frac{\epsilon Q_1(n,m,k)}{H_0^{'}(m,k)}\right](R_m + \cos\theta_k t) \quad (2.92)$$

Setting $\epsilon = 0$ in equation (2.92) above, the result will be solution corresponding to the case in which the inertia effect of the mass of the system is regarded as negligible, then the solution of equation (2.92) becomes

$$Y_{mf}(m,t) = A_{mf}\cos(\alpha_{mf}t - \phi) \tag{2.93}$$

where $A_{mf}, \alpha_{mf}\, and\, \phi_{mf}$ are constants.

Furthermore as $\epsilon < 1$, strubble's technique required that the asymptotic solution of the homogeneous part of the equation (2.92) be of the form

$$Y_m(t) = \Omega(m,t)\cos\Big[\alpha_{mf1}t - \phi(m,t)\Big] + \epsilon Y_1(t) + 0(\epsilon^2) \tag{2.94}$$

where $\Omega(m,t)\, and\, \phi(m,t)$ are slowly varying functions of time or equivalently.

The variational equations describing the behavior of $\Omega(m,t)\, and\, \phi(m,t)$ during the motion of the force are obtained by substituting (2.94) into the homogeneous part of (2.92). Thus, we have

$$\left\{2\phi(\dot{m},t)\alpha_{mf1} - \alpha^2_{mf1} + \frac{H_1^{'}(m,k)}{H_0^{'}(m,k)} - \frac{\epsilon Q_1(n,m,k)H_1^{'}(m,k)}{(H_0^{'}(m,k))^2} + \frac{\epsilon c^2 Q_3(n,m,k)}{H_0^{'}(m,k)}\right\}\times$$

$$\Bigg[\phi(m,t)\cos\left[\alpha_{mf1}t - \phi(m,t)\right]\Bigg] - \left\{2\dot{\phi}(m,t) + \frac{2\epsilon c Q_2(n,m,k)\Omega(m,t)}{H_0^{'}(m,k)}\right\}\alpha_{mf1}\sin\left[\alpha_{mf1}t - \phi(m,t)\right]$$

$$+\epsilon Y_1(m,t) + \frac{H_1^{'}(m,k)}{H_0^{'}(m,k)}\epsilon Y_1(m,t) = 0 \tag{2.95}$$

Extracting those terms which contribute to the variational equation to $0(\epsilon)$, we have

$$2\Omega(m,t)\alpha_{mf1}\dot{\phi}(m,t) - \Omega(m,t)\alpha^2_{mf1} - \frac{\epsilon\psi_{1A}^{'}(m,k)}{(H_0^{'}(m,k))^2}\Omega(m,t) + \frac{H_1^{'}(m,k)}{H_0^{'}(m,k)}\Omega(m,t)$$
$$+\frac{\epsilon c^2\psi_{3A}^{'}(m,k)}{H_0^{'}(m,k)}\Omega(m,t) = 0 \tag{2.96}$$

and

$$\dot{\Omega}(m,t)\alpha_{mf1} + \frac{\epsilon c\psi_{2A}^{'}(m,k)}{H_0^{'}(m,k)}\alpha_{mf1}\Omega(m,t) = 0 \tag{2.97}$$

Setting the coefficients of $\sin\Big[\alpha_{mf1}t - \phi(m,t)\Big]$ and $\cos\Big[\alpha_{mf1}t - \phi(m,t)\Big]$ to zero, and integrate the results, one obtains

$$\phi(m,t) = \frac{\left\{\alpha^2_{mf1}(H_0^{'}(m,k))^2 + \epsilon\psi_{1A}^{'}(m,k) + \epsilon c^2\psi_{3A}^{'}(m,k)H_0^{'}(m,k) - H_{10}^{'}(m,k)\right\}t}{2\alpha_{mf1}(H_0^{'}(m,k))^2} + C_m \tag{2.98}$$

and

$$\phi(m,t) = A_0 e^{-\omega_m t} \tag{2.99}$$

where;

$$\omega_m = \frac{\epsilon c\psi_A^{'}(m,k)}{H_0^{'}(m,k)};\; A_0 = e^{C_0};\; H_{10}^{'}(m,k) = H_1^{'}(m,k)(H_0^{'}(m,k)) \tag{2.100}$$

and $A_0$, $\omega_m$, $C_0$ are constants. Therefore when the effect of the moving distributed mass is considered

,the first approximation to the homogeneous system is given as

$$Y_m(t) = A_0 e^{-\omega_m t} \cos\left[\alpha_{mf1} t - \left\{ \frac{[\alpha_{mf1}^2 (H_0'(m,k))^2 + \epsilon\psi_{1A}'(m,k) + \epsilon c^2 \psi_{3A}'(m,k) H_0'(m,k) - H_{10}'(m,k)]t}{2\alpha_{mf1}(H_0'(m,k))^2} + C_m \right\}\right] \tag{2.101}$$

equation (2.101) implies

$$Y_m(t) = A_0 e^{-\omega_m t} \cos\left[\alpha_{mm} t - C_m\right] \tag{2.102}$$

where;

$$\alpha_{mm} = \alpha_{mf1}\left[\frac{1}{2} - \frac{\epsilon(\psi_{1A}'(m,k) + c^2\psi_{3A}(m,k)H_0'(m,k)) - H_{10}'(m,k)}{2\alpha_{mf1}^2 (H_0'(m,k))^2}\right] \tag{2.103}$$

is called the modified natural frequency, representing the frequency of the system due to the presence of moving mass. Thus to solve the non-homogeneous equation (2.45), the differential operator which acts on $Y_m(t)$ is replaced by the modified frequency $\alpha_{mm}$ to take the form

$$\ddot{Y}_m(t) + \alpha_{mm1}^2 Y_m(t) + \alpha_a Z_m(t) = q_f\left(R_m + \cos\theta_k(t)\right) \tag{2.104}$$

and

$$\ddot{Z}_m(t) + \alpha_{mm2}^2 Z_m(t) - \alpha_b Y_m(t) = 0 \tag{2.105}$$

where;

$$\alpha_{mm1} = \alpha_{mf1}\left[\frac{1}{2} - \frac{\epsilon(\psi_{1A}'(m,k) + c^2\psi_{3A}(m,k)H_0'(m,k)) - H_{10}'(m,k)}{2\alpha_{mf1}^2 (H_0'(m,k))^2}\right] \tag{2.106}$$

$$\alpha_{mm2}^2 = \frac{H_8'(m,k)}{H_7'(m,k)}\ ; \qquad \alpha_a = \frac{H_6'(m,k)}{H_0'(m,k)} \tag{2.107}$$

$$\alpha_b = \frac{H_9'(m,k)}{H_7'(m,k)}\ ; \qquad R_m = -(-1)^m; \qquad \theta_k = \frac{k\pi c}{L} \tag{2.108}$$

These equations (2.104) and (2.105) are analogous to equations (2.56) and (2.57). Thus, using the same argument as in the previous section, one obtains

$$W(x,t) = \sum_{m=1}^{\infty} \frac{\alpha_b Q_f}{\omega_{mm1}^2 \omega_{mm2}^2 (\omega_{mm1}^2 - \omega_{mm2}^2)\omega_{a\theta}^2 \omega_{b\theta}^2} \Big\{ R_m \omega_{a\theta}^2 \omega_{b\theta}^2 \Big[\omega_{mm2}^2 \cos\omega_{mm1} t - \omega_{mm1}^2 \cos\omega_{mm2} t$$
$$+(\omega_{mm1}^2 - \omega_{mm2}^2)\Big] + \omega_{mm1}^2 \omega_{mm2}^2 \Big[\omega_{a\theta}^2 (\cos\theta_k t - \cos\omega_{mm2} t) - \omega_{b\theta}^2 (\cos\theta_k t - \cos\omega_{mm1} t)\Big]\Big\} \cos\frac{m\pi x}{L} \tag{2.109}$$

and

$$V(x,t) = \sum_{m=1}^{\infty} \frac{Q_f}{\omega_{mm1}^2 \omega_{mm2}^2 \omega_{mmd}^2 \omega_{a\theta}^2 \omega_{b\theta}^2} \Big\{ R_m \omega_{a\theta}^2 \omega_{b\theta}^2 \Big[\omega_{mm2}^2 \Big(\omega_{mm1}^2 \cos\omega_{mm2} t - \alpha_{mm2}^2 \cos\omega_{mm1} t\Big)$$
$$-\omega_{mm1}^2 \Big(\omega_{mm2}^2 \cos\omega_{mm1} t + \alpha_{mf2}^2 \cos\omega_{mm2} t\Big) + \alpha_{mm2}^2 \Big(\omega_{mm1}^2 - \omega_{mm2}^2\Big)\Big] + \omega_{mm1}^2 \omega_{mf2}^2 \Big(\omega_{mm1}^2$$
$$-\alpha_{mm2}^2\Big)\Big[\omega_{b\theta}^2 (\cos\theta_k t - \cos\omega_{mm1} t) - \omega_{a\theta}^2 (\cos\theta_k t - \cos\omega_{mm2} t)\Big]\Big\} \sin\frac{m\pi x}{L} \tag{2.110}$$

Equations (2.109) and (2.110) represent the angular displacement and transverse displacement of the thick beam under the action of moving distributed mass for the dynamic problem.

# 3 Numerical Calculation and Discussion of Result

In order to illustrate the analysis in view, the uniform beam of length L=17.5m is considered. The load velocity $c = 30ms^{-1}$ ,Young modulus $E = 2.02 * 10^{11} N/m^3$, moment of inertia $I = .0012m^4$, $\pi = 22/7$, cross sectional area A=7.175 , density of the beam $\rho = 2400kgm^{-3}$, shear coefficient $K^* = 5/6$, shear modulus $G = 7.7E * 10^{10} Nm^{-2}$ and the gravitational acceleration $g = 9.8ms^{-2}$. The transverse displacement and angular displacement of the beam are calculated and plotted against time for various values of axial force N with different values of length "L" of the beam. The results are as shown on the various graphs below.

The effects of the following to the dynamic response of the present beam were investigated:

1. axial force N when length L = 17.5
2. axial force N when length L = 20.0
3. axial force N when length L = 22.5

*Effect of Axial force on the dynamic response when L = 17.5, L = 20 and L = 22.5 for deflection of the dynamic problem*

The effect of axial force on the dynamic response of the uniform simply supported thick beam under distributed moving load for both moving force and moving mass problems are investigated. It is observed that as the axial force is increasing, the amplitude of deflection for both moving force and moving mass problems are decreasing, and vise versa. The results are presented in fig. (2a) to (8b).

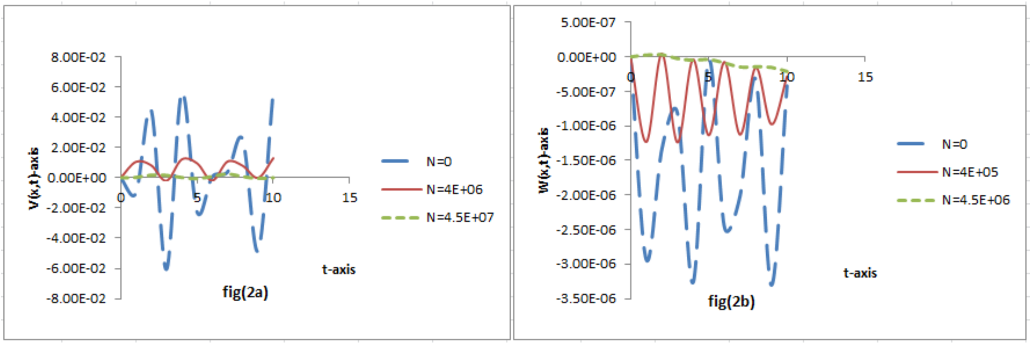


**Fig. 2. The transverse displacement and rotation responses of a thick beam under the action of moving distributed force for various values of axial force when L=17.5**

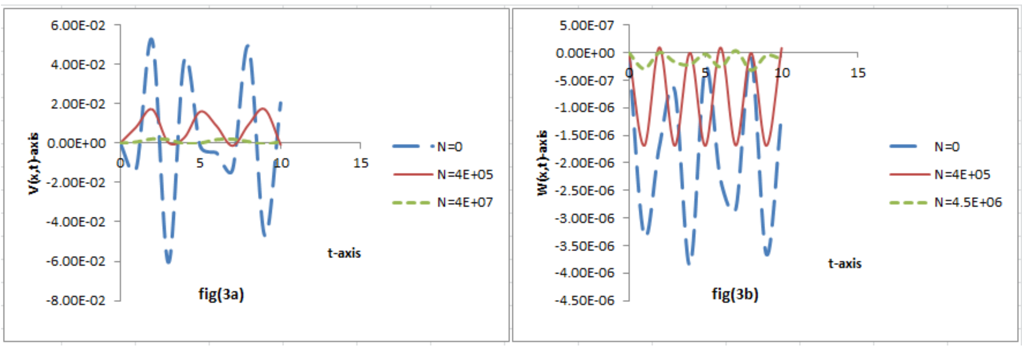


**Fig. 3. The transverse displacement and rotation responses of a thick beam under the action of moving distributed force for various values of axial force when L=20**

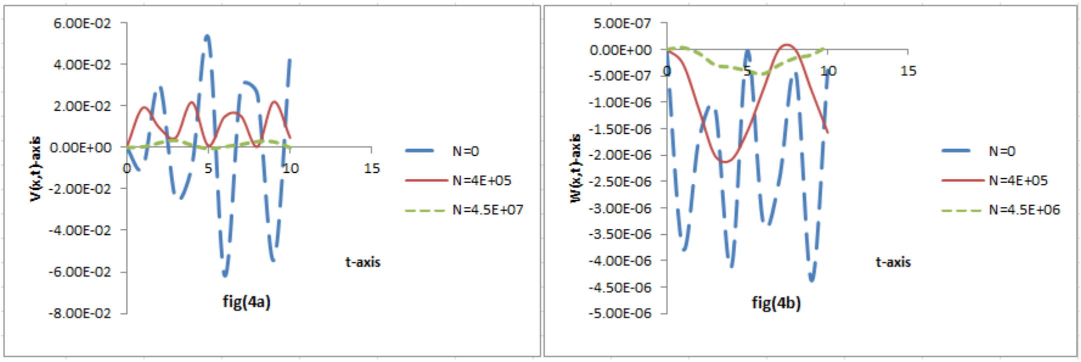


**Fig. 4. The transverse displacement and rotation responses of a thick beam under the action of moving distributed force for various values of axial force when L=22.5**

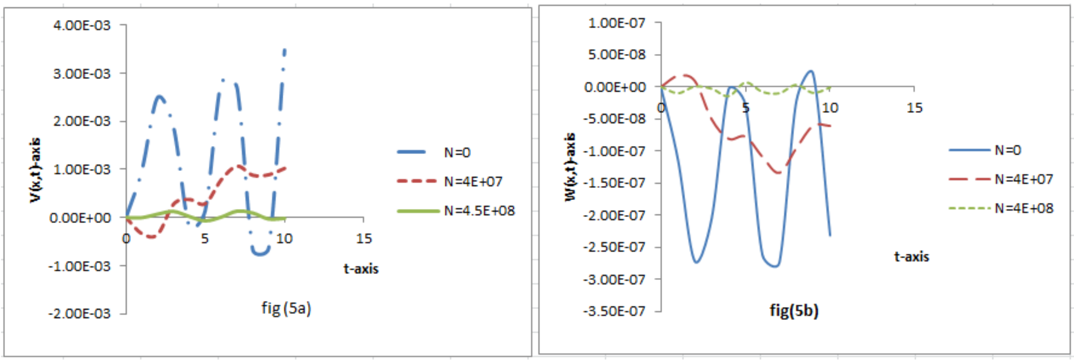


**Fig. 5. The transverse displacement and rotation responses of a thick beam under the action of moving distributed mass for various values of axial force when L=17.5**

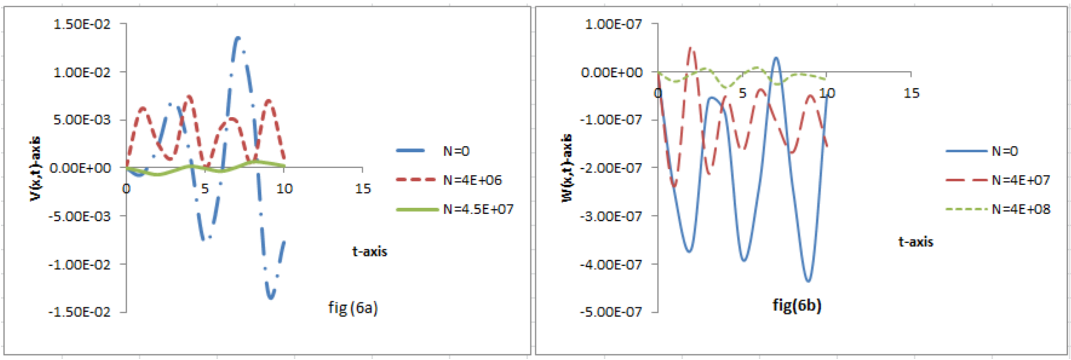


**Fig. 6. The transverse displacement and rotation responses of a thick beam under the action of moving distributed mass for various values of axial force when L=20**

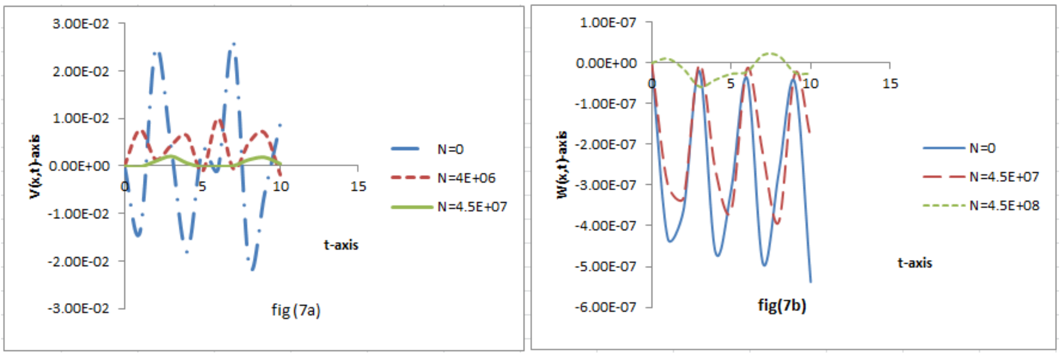


**Fig. 7. The transverse displacement and rotation responses of a thick beam under the action of moving distributed mass for various values of axial force when L=22.5**

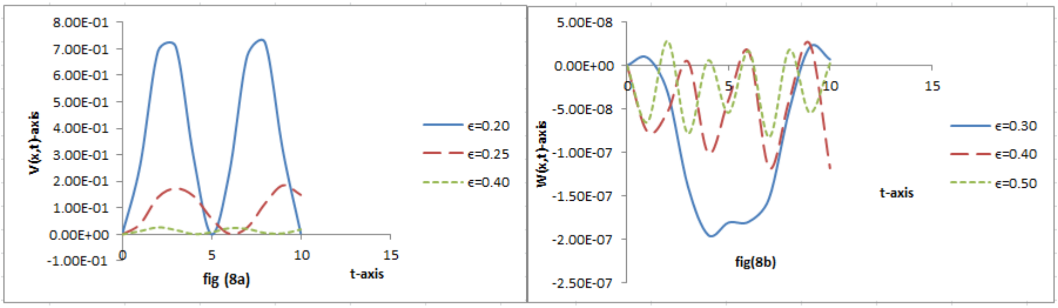


**Fig. 8. The transverse displacement and rotation responses of a thick beam under the action of moving distributed mass for various values of $\epsilon$**

# 4 Conclusion

The effect of variable axial force on the deflection of thick beam under moving load was investigated in this paper. The dynamic problem which is a simply supported has been transformed to a sequence of second order simultaneous ordinary differential equations using Galerkin's method. Thereafter, a modification of Struble's asymptotic method is used to solve the moving mass equation. We have shown that as the prestress is increasing, there exist gradual decrease in the dynamic responses of the beam at different length, an increasing in the value of $\epsilon$, decreases the mode frequency of the thick beam under the action of moving distributed masses for the dynamic system. of the beam for both moving force and moving mass problems.

## Competing Interests

Authors have declared that no competing interests exist.

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