British Journal of Mathematics & Computer Science
14(3): 1-18, 2016, Article no.BJMCS.23247
ISSN: 2231-0851

SCIENCEDOMAIN *international*
www.sciencedomain.org

The Classification of Alternatives Based a Multi-criteria Analysis Technique and the Combination of AHP and the Weighted Sum

Chaibi Ghizlane[1*] and El Khomssi Mohammed[1]

[1]University Sidi Mohammed Ben Abdellah, Modeling Laboratory and Scientific Computing, Faculty of Science and Technology, P.O.Box 2202, Road Imouzzer Fez, Morocco.

Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/BJMCS/2016/23247
Editor(s):
(1) Mohamed Rabea Eid Said, Department of Science and Mathematics, Assiut University, Egypt.
Reviewers:
(1) Wojciech Salabun, West Pomeranian University of Technology, Szczecin, Poland.
(2) Giampaolo Luiz Libralon, Federal Institute of Sao Paulo, Brazil.
(3) Tsung-Yu Hsieh, MingDao University, Taiwan.
(4) Kui Liu, Intelligent Fusion Technology, USA.
Complete Peer review History: http://sciencedomain.org/review-history/13197

Method Article

Received: 22nd November 2015
Accepted: 18th January 2016
Published: 6th February 2016

Abstract

The difficulty of the methods of decision aid is what seeking optimal solutions to complex problems in general and for nature multicriteria, or the concept of optimality is no sense in a multicriteria context.

In this paper, we propose a mathematical modelling, in the context of multicriteria multi- decision-makers aid methods, with a decision-making process allowing the decision-makers to choose the most appropriate solutions according to their orientations, an independent manner, taking into account the objective of the problem.

After identifying the objective and determining the set of actions, we decompose the objective to different dimensions, which gives a system of criteria to different levels.

We use the principle of the AHP method to calculate different weight, related to our decomposition. Also the weighted sums method helps us transform the multi-criteria problem to a mono criterion problem, and obtain a mathematical expression of actions evaluation on which each decision maker bases to choose the optimal action. Therefore, we get a partial order of potential actions on each dimension, and a global order examining all the criteria of the studied system. Which allows to mathematically judge the selection of an action rather than another.

**Corresponding author: E-mail: Ghizlane.CHAIBI@usmba.ac.ma, khomsixmath@yahoo.fr;*

Finally, we start with the data of the company Rabat-Sale tramway and apply the proposed process to judge the choice of the tram as a means of transport instead of the bus.
This is a concrete example of large financial size showing the effectiveness of our proposal.

Keywords: AHP method; alternative; classification; decision maker; multi-criteria analysis; weighted ratings.

1 Introduction

The mathematical methods of multi-criterion analysis allowed to direct a choice on the basis of several common criteria, having identified and defined the problem, by nature, has several objectives, often contradictory or heterogeneous. To have several decision-makers diversified the set of possible objectives for the same problem, and the same decision-maker can have several objectives. Every objective can lead several criteria, and consequently of the creation of several measures of evaluations. Which creates the difficulty to find a general consensus within a multidisciplinary team.

In this work we consider a multi-criteria multi-decision maker problem (MCMD) as a set of sub-multi-criteria mono-decision maker problems (MCSD). The criteria in each sub-multi-criteria mono- decision maker problem are not necessarily the same. Using this analysis, the team members do not need to agree on the relative importance of the criteria or on the ranking of alternatives. In other words, each decision maker has the possibility to choose a set of criteria and gives his own judgment from his set of selected criteria and contributes in a separate and identifiable manner to the search for a joint conclusion.

The notion of time makers the problem dynamic, instead of admitting that the function of evaluation of a criterion depends only on the action, we define it in a way which guarantees an accurate information at a moment t, because the objective is to help the decision-makers to make the best decision among a number of possibilities. Then we use the principle of the AHP method to affect three types of weights [1]; the weight of the dimension, the relative weight of the criterion, then its global weight.

We inject the procedure for calculating weighted sums in each dimension, and we use the relative weights of the criteria with instant evaluations to find the best action in relation to the dimension. When we seek the optimal action for the global objective, we use these obtained sums to define the global weighted sums.

In numerous situations, the nature of the actions does not allow us to have, each time, the criteria evaluations, this the reason why we introduce what we call the average weighted sums. We calculate these sums to find the best solution, which also gives strategies of sequencing of the involved actions.

2 Identification of the Dimensions and the Criteria

2.1 Decomposition of the objective

When n decision-makers [2] seek the optimal solution for a problem or a situation, they need to identify the objective to achieve noted O, and a set of alternatives noted A such as:

$$A = \{a_j : j = 1,2, \dots, m\}$$

It can be discrete or continuous, and it is also the same for all decision- makers[1].

This is a set of m alternatives, which represent the decision object, the intention is to identify a subset of alternatives with a better compromise among the set starting.

[1] *The set A is defined via the nature of the problem studied, so it is the same for all decision makers.*

For this purpose, we introduce the notion of optimality of alternative [3,4] through the following definition.

Definition: An optimal alternative is any preferred alternative by the decision maker on the basis of external information to the mathematical model.

The solution of the decision problem amounts to search of a partition of the set of possible alternatives in a subset of "optimal" alternatives and a subset of "non-optimal" alternatives.

Let's note D the set of decision-makers, it as in the form:

$$D = \{D_i : i = 1, 2, \ldots, n\}$$

each D_i decision maker examines this objective via the different independent dimensions (economic, social, political ...) noted $d_{i,k}$, consequently, the set of the dimensions selected by the decision maker D_i is:

$$\widetilde{D}_i = \{d_{i,k} : k = 1, 2, \ldots, l_i\}$$

with l_i is the number of dimensions chosen by the decision maker D_i.

So the objective O is considered as a decomposable set to sub-objectives [5] independent noted $O_{i,k}$ (the restriction of the objective O of the dimension $d_{i,k}$: $(O_{i,k} = O / d_{i,k})$, and we have for each D_i decision maker:

$$O = \bigcup_{k=1}^{l_i} O_{i,k} \tag{1}$$

The objective $O_{i,k}$ related to the dimension $d_{i,k}$ is achieved through the minimization or maximization of a set of criteria, So the decision maker has to determine a set of criteria noted $C_{i,k,p}$ which measures the effectiveness of actions [6]. This set is written as:

$$C_{i,k} = \{C_{i,k,p} : p = 1,2, \ldots, p_{i,k}\}$$

with $C_{i,k,p}$ is a criterion chosen by the decider D_i in the dimension $d_{i,k}$.

The number $p_{i,k} = \mathrm{card}(C_{i,k})$ is the number of criteria in dimension $d_{i,k}$, it will vary from one decision-maker to another and from one dimension to another.

This union $C_i = \bigcup_{k=1}^{l_i} C_{i,k}$ is the set[2] of all the criteria considered by decision-maker D_i.

The maximization or the minimization of $C_{i,k,p}$ is a sub- objective $O_{i,k,p}$, therefore:

$$O_{i,k} = \bigcup_{p=1}^{p_{i,k}} O_{i,k,p} \tag{2}$$

Which gives:

$$O = \bigcup_{k=1}^{l_i} \bigcup_{p=1}^{p_{i,k}} O_{i,k,p} \tag{3}$$

[2] *This union is disjoint by construction.*

Thus, we construct a cover partition of the overall objective[3].

Definition: Let O an objective, $(O_k)_{k=1,..,n}$ a finite sequence of objectives constituting a partition of O. Each objective O_k is linked to a dimension d_k weight of P_k.

A dimension d_{k_0} is called negligible[4] if and only if P_{k_0} tends to 0, in other words

$$\sum_{k\neq k_0}^{n} P_k \cong 1$$

In such a modelling, it is possible that the decision-maker chose a set of essential objectives $(O_{i,k})_{k=1,..,l_i}$ for his modeling without covering the overall objective of the study, i.e

$$O = \bigcup_{k=1}^{l_i} O_{i,k} \bigcup \tilde{O}_i \tag{4}$$

Its modeling is acceptable if and only if the dimension $\tilde{d}_i$ for objective $\tilde{O}_i$ is negligible in the previously defined sense.

In the following, this modeling without losing the generality, we simply write $O = \bigcup_{k=1}^{l_i} O_{i,k}$

Fig. 1 reflects this idea of decomposition of the objectives.

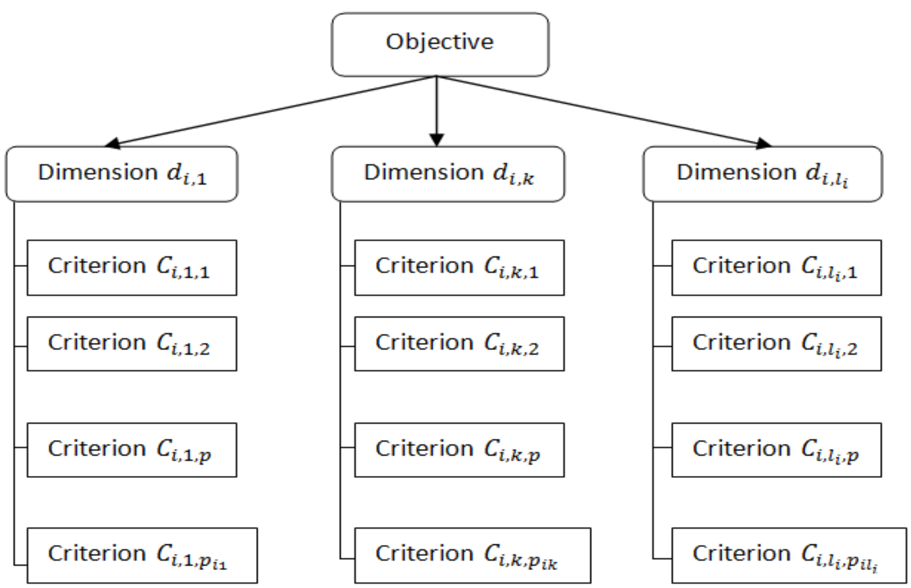

Fig. 1. Decomposition of the objective

[3] *The objectives $O_{i,k}$ can be grouped into two classe, one containing the objectives realized in parallel, other containing the objectives realized series.*

[4] *Say $P_{i_o} \cong 0$ is strongly connected with context of the subject and preferences of the decision maker. They specify the neighborhood considered in the study of zero. i.e each study has a threshold from which we talk about of a negligible dimension.*

2.2 Evaluation and compatibility of criteria

Let T the set of time, we have two possibilities:

- T is a discrete time. It is written in the form[5]

  $T = \{\alpha h\colon\ \alpha \in \mathbb{Z}, h\ \ a\ fixed\ positive\ constant\}$

- T is a continuous time, thus $T = \mathbb{R}$ or $T = \mathbb{R}^+$.

So we define the functions of evaluations as follows [7].

Definition: For each criterion $C_{i,k,p}$, we define the evaluation[6] function.

$$g_{ikp} : T \times A \rightarrow \mathbb{R}$$
$$g_{ikp}\left(t, a_j\right) = \mathrm{e_{i,kpj}(t)} \quad (5)$$

From these functions, decision-maker built an evaluation set at the moment t denote $E_i(\mathrm{t})$ is defined as follows:

$$E_i(\mathrm{t}) = \left\{\mathrm{e_{i,kpj}(t)}\colon \mathrm{k} = 1,2, \ldots, l_i \quad \mathrm{p} = 1,2, \ldots, p_{i,k} \quad \mathrm{j} = 1,2, \ldots, \mathrm{m}\right\}$$

It's the set of evaluation for decision- marker D_i at instant t.

Remark: The evaluation function $g_{i,k,p}$ is related to the criterion $C_{i,k,p}$ i.e the criterion p selected in the dimension k by decision–maker D_i , therefore this function is written in relation to the criterion and not in relation to the decision-maker. The index i is just to say that the criterion is chosen by the decider D_i .

As an example, In the case of a comparative multi-criteria analysis of transport systems in a specific region, for a decision-maker D_i , the speed can be considered as a criterion of performance dimension noted $C_{i,k,p}$. For another decision-maker $D_{i'}$ it can be considered as a criterion of degree of technology dimension noted $C_{i',k',p'}$, and we will necessarily have $\mathrm{e_{i,kpj}(t)} = \mathrm{e}_{i',k'p'\mathrm{j}}(\mathrm{t})$ for each alternative j.

We have seen previously that the achievement of sub-objectives related to the dimension depends on the maximization or minimization of criteria. In this approach, we want to use weighted sums, to have a coherence in our study. It is necessary to consider only the criteria to be maximized, but criteria to minimize exist (the criteria related to the cost are the criteria to minimize). A transformation of these data is then necessary to obtain those criteria to maximize. The transformation is as follows:

$$e'_{i,kpj}(t) = \max_{\mathrm{j}} e_{i,kpj}(t) - e_{i,kpj}(t) \quad (6)$$

For each criteria p and dimension k.

In what follows, we assume that there are criteria only to maximize, because it is always possible to turn the objective of minimizing an objective of maximizing through our writing. Thus, for each decision maker, we summarize the data in the following table:

---

[5] The parameter h is the step of discretization time and the parameter α exchange of sign for model the problem in the past and the present.

[6] The properties of function g_{ikp} depends on the context of the probleme, this last imposes the form of the spaces T and A (continuous or discrete) ... and even the nature of the criteria.

Table 1. Evaluation of decision–marker D_i

Dimension		$d_{i,1}$			$\cdots$	$d_{i,k}$			$\cdots$	d_{i,l_i}		
Criterion		$C_{i,1,1}$	$\cdots$	$C_{i,1,p_{i1}}$	$C_{i,k,1}$	$\cdots$	$C_{i,k,p_{ik}}$		$C_{i,l_i,1}$	$\cdots$	$C_{i,l_i,p_{il_i}}$	
Alternative	a_1	$e_{i,111}(t)$		$e_{i,1p_{i1}1}(t)$	$e_{i,k11}(t)$		$e_{i,kp_{ik}1}(t)$		$e_{i,l_i,11}(t)$		$e_{i,l_i,p_{il_i}1}(t)$	
	a_2	$e_{i,112}(t)$		$e_{i,1p_{i1}2}(t)$	$e_{i,k12}(t)$		$e_{i,kp_{ik}2}(t)$		$e_{i,l_i,12}(t)$		$e_{i,l_i,p_{il_i}2}(t)$	
	$\vdots$	$\vdots$										
	a_j	$e_{i,11j}(t)$		$e_{i,1p_{i1}j}(t)$	$e_{i,k1j}(t)$		$e_{i,kp_{ik}j}(t)$		$e_{i,l_i,1j}(t)$		$e_{i,l_i,p_{il_i}j}(t)$	
	$\vdots$	$\vdots$										
	a_m	$e_{i,11m}(t)$		$e_{i,1p_{i1}m}(t)$	$e_{i,k1m}(t)$		$e_{i,kp_{ik}m}(t)$		$e_{i,l_i,1m}(t)$		$e_{i,l_i,p_{il_i}m}(t)$	

3 Determination of Weight

3.1 The weight dimensions

The transcript of the AHP method introduced by Saaty [8], allows us to build the weights of the dimensions. Indeed, the decision-maker compares these dimensions between them, two for two, using the weighting scale presented in the following table.

Table 2. The saaty rating scale

Intensity of importance	Definition	Explanation
1	Equal importance	Two factors contribute equally to the objective.
3	Somewhat more important	Experience and judgement slightly favour one over the other.
5	Much more important	Experience and judgement strongly favour one over the other.
7	Very much more important	Experience and judgement very strongly favour one over the other. Its importance is demonstrated in practice.
9	Absolutely more important.	The evidence favouring one over the other is of the highest possible validity.
2, 4, 6, 8	Intermediate values	When compromise is needed.

From this important scale, the decision-maker proposes the matrix of comparison between these chosen dimensions $d_{i,k}$ and $d_{i,\tilde{k}}$ following:

$$\begin{pmatrix} 1 & v_{i,12} & \cdots & v_{i,1k} & \cdots & v_{i,1l_i} \\ 1/v_{i,12} & 1 & \cdots & v_{i,2k} & \cdots & v_{i,2l_i} \\ \vdots & \vdots & 1 & \vdots & \vdots & \vdots \\ 1/v_{i,1k} & 1/v_{i,2k} & \cdots & 1 & \cdots & v_{i,kl_i} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ 1/v_{i,1l_i} & 1/v_{i,2l_i} & \cdots & 1/v_{i,kl_i} & \cdots & 1 \end{pmatrix}$$

Comparision matrix of dimensions

It is a square matrix $(v_{i,k\tilde{k}})_{k,\tilde{k}=1,\ldots,l_i}$ such as:

- The coefficients $v_{i,kk}$=1, because it compares the dimension with herself.
- For $k \neq \tilde{k}$, if $d_{i,k}$ is $v_{i,k\tilde{k}}$ times important then $d_{i,\tilde{k}}$ then $d_{i,\tilde{k}}$ is $\frac{1}{v_{i,k\tilde{k}}}$ times important then $d_{i,k}$, therefore coefficients of the comparison matrix verify the relation $v_{i,\tilde{k}k} = \frac{1}{v_{i,k\tilde{k}}}$.

With the matrix the decision-maker calculates $P_{i,k}$ associated at the dimension $d_{i,k}$ as follows:

$$P_{i,k} = \frac{1}{l_i}\left(\sum_{m=1}^{l_i} \frac{v_{i,km}}{\sum_{j=1}^{l_i} v_{i,jm}}\right) \tag{7}$$

3.2 Relative weights of the criteria

After calculation with weight of the dimensions $d_{i,k}$ $k: 1, 2, \dots, l_i$, the decision-maker have to introduce a square matrix of order $p_{i,k}$, wherein the decider compares the criteria of a dimension indicating the importance of $C_{i,k,p}$ with $C_{i,k,p'}$ from the scale of Saaty. The following matrix illustrates this comparison:

$$\begin{pmatrix} 1 & \tilde{v}_{ik,12} & \cdots & \tilde{v}_{ik,1p} & \cdots & \tilde{v}_{ik,1p_{ik}} \\ 1/\tilde{v}_{ik,12} & 1 & \cdots & \tilde{v}_{ik,2p} & \cdots & \tilde{v}_{ik,2p_{ik}} \\ \vdots & \vdots & 1 & \vdots & \vdots & \vdots \\ 1/\tilde{v}_{ik,1p} & 1/\tilde{v}_{ik,2p} & \cdots & 1 & \cdots & \tilde{v}_{ik,pp_{ik}} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ 1/\tilde{v}_{ik,1p_{ik}} & 1/\tilde{v}_{ik,2p_{ik}} & \cdots & 1/\tilde{v}_{ik,pp_{ik}} & \cdots & 1 \end{pmatrix}$$

Comparision matrix of criteria

The relative weight $P_{i,k,p}$ associated with the criterion $C_{i,k,p}$ is defined as:

$$P_{i,k,p} = \frac{1}{p_{ik}}\left(\sum_{j=1}^{p_{ik}} \frac{\tilde{v}_{ik,pj}}{\sum_{l=1}^{p_{ik}} \tilde{v}_{ik,lj}}\right) \tag{8}$$

The AHP method guaranteed the fact that:

$$\sum_{p=1}^{p_{ik}} P_{i,k,p} = 1$$

3.3 Global weight of the criteria

After the construction of weights $P_{i,k}$ of dimensions $d_{i,k}$ and $P_{i,k,p}$ relative weights of the criteria $C_{i,k,p}$, we define the global weights of the criteria $C_{i,k,p}$ denoted $w_{i,k,p}$ as follows:

Definition: For each criterion $C_{i,k,p}$ in relation to the dimension $d_{i,k}$, the global weight is defined by:

$$w_{i,k,p} = P_{i,k}P_{i,k,p} \tag{9}$$

This weight is called global because it presents the importance of the criterion $C_{i,k,p}$ in relation to all the criteria of the structure. Moreover, the weight of the dimension is the sum of the overall weight of all criteria, indeed:

$$\sum_{p=1}^{p_{ik}} w_{i,k,p} = \sum_{p=1}^{p_{ik}} P_{i,k}P_{i,k,p} = P_{i,k}\sum_{p=1}^{p_{ik}} P_{i,k,p} = P_{i,k}$$

This is natural, since the weight of a group is defined by the sum of the weights of elements forming this group.

The following table illustrates the different weights built by decision- marker D_i in relation to the dimension $d_{i,k}$:

Table 3. Different weight related to dimension

	Dimension $d_{i,k}$					
Weights				$P_{i,k}$		
Criterion	$C_{i,k,1}$	$C_{i,k,2}$	…	$C_{i,k,p}$	…	$C_{i,k,p_{ik}}$	**Sum**
Relative weight	$P_{i,k,1}$	$P_{i,k,2}$	…	$P_{i,k,p}$	…	$P_{i,k,p_{ik}}$	$\sum_{p=1}^{p_{ik}} P_{i,k,p} = 1$
Overall weight	$w_{i,k,1}$	$w_{i,k,2}$	…	$w_{i,k,p}$	…	$w_{i,k,p_{ik}}$	$\sum_{p=1}^{p_{ik}} w_{i,k,p} = P_{i,k}$

At this point, we decomposed the objective of different dimensions, which are decomposed to a set of criteria that are measured with their evaluation functions. We also calculated the different weights, the following table lists the different information by decision-marker:

Table 4. The data necessary for resolution

Dimension		$d_{i,1}$		⋯	$d_{i,k}$	⋯	d_{i,l_i}	
Weight		$P_{i,1}$			$P_{i,k}$		P_{i,l_i}	
Criterion		$C_{i,1,1}$ ⋯	$C_{i,1,p_{i1}}$	$C_{i,k,1}$ ⋯	$C_{i,k,p_{ik}}$	$C_{i,l_i,1}$ ⋯	$C_{i,l_i,p_{il_i}}$	
Relative weight		$P_{i,1,1}$	$P_{i,1,p_{i1}}$	$P_{i,k,1}$	$P_{i,k,p_{ik}}$	$P_{i,l_i,1}$	$P_{i,l_i,p_{il_i}}$	
Overall weight		$w_{i,1,1}$	$w_{i,1,p_{i1}}$	$w_{i,k,1}$	$w_{i,k,p_{ik}}$	$w_{i,l_i,1}$	$w_{i,l_i,p_{il_i}}$	
Alternative	a_1	$e_{i,111}(t)$	$e_{i,1p_{i1}1}(t)$	$e_{i,k11}(t)$	$e_{i,kp_{ik}1}(t)$	$e_{i,l_i,11}(t)$	$e_{i,l_i,p_{il_i}1}(t)$	
	a_2	$e_{i,112}(t)$	$e_{i,1p_{i1}2}(t)$	$e_{i,k12}(t)$	$e_{i,kp_{ik}2}(t)$	$e_{i,l_i,12}(t)$	$e_{i,l_i,p_{il_i}2}(t)$	
	⋮	⋮						
	a_j	$e_{i,11j}(t)$	$e_{i,1p_{i1}j}(t)$	$e_{i,k1j}(t)$	$e_{i,kp_{ik}j}(t)$	$e_{i,l_i,1j}(t)$	$e_{i,l_i,p_{il_i}j}(t)$	
	⋮	⋮						
	a_m	$e_{i,11m}(t)$	$e_{i,1p_{i1}m}(t)$	$e_{i,k1m}(t)$	$e_{i,kp_{ik}m}(t)$	$e_{i,l_i,1m}(t)$	$e_{i,l_i,p_{il_i}m}(t)$	

4 Calculation of the Weighted Sums

After assigning weights to the criteria, and the evaluation criteria for each alternative, we calculate the weighted ratings for the alternative using the principle of the weighted sum method [9].

If the decision-maker D_i seeks the optimal alternative, he has at least two possibilities to consider, which are cited in the following paragraphs.

4.1 Relative weighted sum

If the decision maker D_i is interested in a precise dimension $d_{i,k}$, he can calculate the time t on the weighted sum denoted $S_{i,k}$ for any alternative as follows, using the criteria $C_{i,k,p}$ and its weight relative $P_{i,k,p}$ quite consider that criteria to be maximized:

$$S_{i,k}(t, a_j) = \sum_{p=1}^{p_{ik}} P_{i,k,p}\, g_{ikp}(t, a_j) \quad (10)$$

Or

$$S_{i,k}(t,a_j) = \sum_{p=1}^{p_{ik}} P_{i,k,p}\, e_{i,kpj}(t) \tag{11}$$

Which provides the table of results as follows:

Table 5. Relative weighted sum

		Dimension $d_{i,k}$						
Weights		**$P_{i,k}$**						
Criterion		$C_{i,k,1}$	$C_{i,k,2}$	...	$C_{i,k,p}$	...	$C_{i,k,p_{ik}}$	Sum
Relative weight		$P_{i,k,1}$	$P_{i,k,2}$	...	$P_{i,k,p}$	...	$P_{i,k,p_{ik}}$	$\sum_{p=1}^{p_{ik}} P_{i,k,p} = 1$
Alternative	a_1	$e_{i,k11}(t)$	$e_{i,k21}(t)$	...	$e_{i,kp1}(t)$	...	$e_{i,kp_{ik}1}(t)$	$S_{i,k}(t,a_1)$
	a_2	$e_{i,k12}(t)$	$e_{i,k22}(t)$		$e_{i,kp2}(t)$		$e_{i,kp_{ik}2}(t)$	$S_{i,k}(t,a_2)$
	$\vdots$							
	a_j	$e_{i,k1j}(t)$	$e_{i,k2j}(t)$		$e_{i,kpj}(t)$		$e_{i,kp_{ik}j}(t)$	$S_{i,k}(t,a_j)$
	$\vdots$							
	a_m	$e_{i,k1m}(t)$	$e_{i,k2m}(t)$		$e_{i,kpm}(t)$		$e_{i,kp_{ik}m}(t)$	$S_{i,k}(t,a_m)$

This transforms the problem to a problem with a single criterion, and the looked optimal alternative a_{j_0} at time t is the One that is satisfactory:

$$S_{i,k}(t,a_{j_0}) = \max_j S_{i,k}(t,a_j) \tag{12}$$

4.2 Global weighted sum

If the decision-maker D_i is looking at instant t optimal alternative to achieve the global objective, it must consider all dimensions, and the criteria defined from on relative weighted sum. He should consider the following table of data:

Table 6. Evaluation with relative weighted sum

Dimension			**$d_{i,1}$**	**$d_{i,1}$**	⋯	**$d_{i,k}$**	⋯	**d_{i,l_i}**
Weight			**$P_{i,1}$**	**$P_{i,1}$**		**$P_{i,k}$**		**P_{i,l_i}**
Alternative	a_1	Relative Weighted Sum	$S_{i,1}(t,a_1)$	$S_{i,2}(t,a_1)$		$S_{i,k}(t,a_1)$		$S_{i,l_i}(t,a_1)$
	a_2		$S_{i,1}(t,a_2)$	$S_{i,2}(t,a_2)$		$S_{i,k}(t,a_2)$		$S_{i,l_i}(t,a_2)$
	$\vdots$							
	a_j		$S_{i,1}(t,a_j)$	$S_{i,2}(t,a_j)$		$S_{i,k}(t,a_j)$		$S_{i,l_i}(t,a_j)$
	$\vdots$							
	a_m		$S_{i,1}(t,a_m)$	$S_{i,2}(t,a_m)$		$S_{i,k}(t,a_m)$		$S_{i,l_i}(t,a_m)$

From these data, we define the global weighted sum as follows:

Definition: For each decision maker D_i and any alternative a_j at any time t we define the global weighted sum denoted $S_i(t,a_j)$ as follows:

$$S_i(t,a_j) = \sum_{k=1}^{l_i} P_{i,k}\, S_{i,k}(t,a_j) \tag{13}$$

Using this definition the optimal alternative $a_{\bar{j}}$ is the one that satisfying:

$$S_i(t, a_{\bar{j}}) = \max_j S_i(t, a_j) \tag{14}$$

If we replace $S_{i,k}(t, a_j)$ by its expression we get:

$$S_i(t, a_j) = \sum_{k=1}^{l_i} P_{i,k}\, S_{i,k}(t, a_j) = \sum_{k=1}^{l_i} P_{i,k} \sum_{p=1}^{p_{ik}} P_{i,k,p}\, e_{i,kpj}(t) = \sum_{k=1}^{l_i} \sum_{p=1}^{p_{ik}} P_{i,k}\, P_{i,k,p}\, e_{i,kpj}(t)$$

Therefore

$$S_i(t, a_j) = \sum_{k=1}^{l_i} \sum_{p=1}^{p_{ik}} w_{i,k,p}\, e_{i,kpj}(t)$$

This formula shows that the weighted total is the sum of the products of the weights of all the criteria on the structure and evaluation of these, so we got the classic formula of the weighted sum, which reinforces our model in the sense that it is more general.

4.3 Average weighted sum

Practically, seeking the optimal action at a time t can return the decision more difficult, especially if that optimal action is changed on small intervals in the case of a continuous time or an instant t_i to t_{i+1} for a discrete time. For this reason, we introduce the notion of average gives a wider vision of the concept of time in both cases (continuous or discrete time).

Definition: We define the weighted average sum in relation to the dimension $d_{i,k}$ denoted $S_{i,k}^m(a_j)$ as follows:

- with a continuous time

$$S_{i,k}^m(a_j) = \frac{1}{t_f - t_i} \sum_{p=1}^{p_{ik}} P_{i,k,p} \int_{t_i}^{t_f} g_{ikp}(t, a_j)\, dt \qquad t_f \neq t_i \tag{15}$$

- With a discrete time

$$S_{i,k}^m(a_j) = \frac{1}{N} \sum_{p=1}^{p_{ik}} P_{i,k,p} \sum_{t=t_i}^{t_f} g_{ikp}(t, a_j) \tag{16}$$

N is the number of steps.

The optimal action $a_{\bar{j}}$ searched between t_i and t_f is one that checks:

$$S_{i,k}^m(a_{\bar{j}}) = \max_j S_{i,k}^m(a_j) \tag{17}$$

Definition: for the decider-maker D_i and alternative a_j, we define the global weighted average sum denoted $S_i^m(a_j)$ as follows:

$$S_i^m(a_j) = \sum_{k=1}^{l_i} P_{i,k} S_{i,k}^m(a_j) \tag{18}$$

The optimal share a_J searched between t_i and t_f is one that checks

$$S_i^m(a_J) = \max_j S_i^m(a_j) \tag{19}$$

We can summarize the principal idea of this modeling and say that it can respond in a comprehensive or selectively manner looking for an optimal action following what we want to maximize.

5 **Classification of Potential Alternatives**

In this section, without losing the generality, we consider that the average weighted sums, that transform the instant study at a study spread over time, allow decision makers to identify the representative periods to study via the nature of the modeled problem. Knowing that it is possible to repeat the same approach presented below for instant study.

5.1 Order related to the dimension $\mathbf{d_{i,k}}$

Let $A_1 = A$, if the decider-maker D_i aim is to order potential alternatives in a specific dimension $d_{i,k}$, obviously the optimal solution is the first action denoted a_{j_1} which verifies:

$$S_{i,k}^m(a_{j_1}) = \max_{a_j \in A_1} S_{i,k}^m(a_j)$$

Let $A_2 = A_1 \setminus \{a_{j_1}\}$, the second alternative denoted a_{J_2}which verifies:

$$S_{i,k}^m(a_{j_2}) = \max_{a_j \in A_2} S_{i,k}^m(a_j)$$

Generally the l^{th} alternative noted alternative a_{j_l} is the one that verifies

$$S_{i,k}^m(a_{j_l}) = \max_{a_j \in A_l} S_{i,k}^m(a_j)$$

with $A_{l+1} = A_l \setminus \{a_{j_l}\}$ for $1 \le l \le m-1$ and $A_m = \{a_{j_m}\}$

Therefore

$$A = \bigcup_{l=1}^{m} \{a_{j_l}\}$$

This is a canonical partition ordered for all potential alternative A in the dimension $d_{i,k}$.

5.2 Global order of alternatives

If the decision maker D_i aim is to order the set of potential alternative relative to the overall objective, it is sufficient to repeat the same algorithm of the previous paragraph, replacing the averages weighted sum are about $S_{i,k}^m(a_j)$ by global averages weighted sum $S_i^m(a_j)$, for obtaining an ordered canonical partition of all potential alternative A relative to the goal studied.

6 Application: Comparison Tram and Bus

6.1 Problematic

The aim of multi-criteria analysis of transportation systems is to seek a way which guarantees quality travel conditions for citizens to meet different health social, economic, environmental and even health issues and which allows:

- Minimize transportation costs such as the purchase and maintenance of company vehicles.
- Optimize travel, infrastructure.
- Reduce costs related to parking and consequently reduce the costs associated with it.
- Reduce delays.
- Reduce emissions of GHG gases.
- Improved security trips.
- Reduced stress and fatigue of citizens.
- Reducing the number of accidents on roads.

In the result of this work, we will perform a multi-criteria analysis of company data from Rabat-Sale[7] tramway (STRS) and data from the National Office of Hydrocarbons and Mines (ONHYM) in order to evaluate the system Transport Rabat-Sale.

Due to the absence of sufficient data to diversify the dimensions and criteria, we limit ourselves to certain criteria which we have their assessments.

We define the same dimensions and the same criteria for all decision makers, again we do not have the formulas of functions of evaluations that it is private data averages of the company Rabat-Sale tramway.

6.2 Identify the dimensions and criteria

All the criteria in this application will be imposed for all decision makers.

To differentiate a transport system to another, we use the following dimensions and criteria:

Table 7. Identification of dimensions

Dimension $d_{i,1}$	The performance and rendered services
Dimension $d_{i,2}$	Cost
Dimension $d_{i,3}$	Environment

For the first dimension: The performance of transport systems is a composite measure of overall capacity, frequency, commercial speed, reliability and productivity. Punctuality and accessibility measure the quality of services provided to users of transport systems. But we choose the criteria that we have their assessments:

- $C_{i,1,1}$: Capacity
- $C_{i,1,2}$: Frequency
- $C_{i,1,3}$: Commercial speed:
- $C_{i,1,4}$: Punctuality

For the second dimension: In this dimension, we consider a single criterion is:

- $C_{i,2,1}$: Investment costs.

[7] *Moroccan city of North African.*

The investment cost is an estimated cost of landing endorsements markets. This cost includes:

- Study contracts and Tramway works.
- Pre-operating expenses
- Financial expenses (before starting operation)

For the third dimension:

- $C_{i,31}$: Greenhouse Effect gas (GHG)

A GHG gas is a gas that affects climate change. Particularly halocarbons that man artificially manufactured.

6.3 Assign the weights

In this study, the weighting will be performed at two levels:

- The first: three dimensions will weigh between them, which we assign weights to aspects of performance and services rendered, cost and pollution.
- The second: the criteria of a same dimension will weigh them. Now for the second and the third dimension, we have no criteria, therefore we seek the relative weights of the criteria just the first dimension.

Below is the comparison matrix of the dimensions for the four decision makers.

$$\begin{matrix} & d_{1,1} & d_{1,2} & d_{1,3} \\ d_{1,1} & 1 & 3 & 7 \\ d_{1,2} & 1/3 & 1 & 5 \\ d_{1,3} & 1/7 & 1/5 & 1 \end{matrix} \qquad \begin{matrix} & d_{2,1} & d_{2,2} & d_{2,3} \\ d_{2,1} & 1 & 7 & 9 \\ d_{2,2} & 1/7 & 1 & 5 \\ d_{2,3} & 1/9 & 1/5 & 1 \end{matrix} \qquad \begin{matrix} & d_{3,1} & d_{3,2} & d_{3,3} \\ d_{3,1} & 1 & 1/3 & 5 \\ d_{3,2} & 3 & 1 & 7 \\ d_{3,3} & 1/5 & 1/7 & 1 \end{matrix} \qquad \begin{matrix} & d_{4,1} & d_{4,2} & d_{4,3} \\ d_{4,1} & 1 & 3 & 3 \\ d_{4,2} & 1/3 & 1 & 1 \\ d_{1,3} & 1/3 & 1 & 1 \end{matrix}$$

The comparison matrix of the dimensions

Each decision-maker met its comparison matrix from the scale of Saaty to assess the importance of a dimension to another's point of life. Then we calculate the weights of the dimensions from the formula 7, thus we find:

Table 8. Weights of the dimensions

D_1			D_2			D_3			D_4		
$P_{1,1}$	$P_{1,2}$	$P_{1,3}$	$P_{2,1}$	$P_{2,2}$	$P_{2,3}$	$P_{3,1}$	$P_{3,2}$	$P_{1,3}$	$P_{4,1}$	$P_{4,2}$	$P_{4,3}$
0.65	0.28	0.07	0.75	0.2	0.05	0.28	0.65	0.07	0.6	0.2	0.2

Now we look for the relative weights of the criteria of the first dimension to the four makers in an analogous manner to the calculation of the weights of the dimensions. For this, we start with the comparison matrices of the following criteria:

$$\begin{matrix} & C_{1,1,1} & C_{1,1,2} & C_{1,1,3} & C_{1,1,4} \\ C_{1,1,1} & 1 & 5 & 3 & 3 \\ C_{1,1,2} & 1/5 & 1 & 1/3 & 1/3 \\ C_{1,1,3} & 1/3 & 3 & 1 & 1 \\ C_{1,1,4} & 1/3 & 3 & 1 & 1 \end{matrix} \qquad \begin{matrix} & C_{2,1,1} & C_{2,1,2} & C_{2,1,3} & C_{2,1,4} \\ C_{2,1,1} & 1 & 5 & 3 & 3 \\ C_{2,1,2} & 1/5 & 1 & 1/3 & 1/3 \\ C_{2,1,3} & 1/3 & 3 & 1 & 1 \\ C_{2,1,4} & 1/3 & 3 & 1 & 1 \end{matrix}$$

$$\begin{array}{c} \\ C_{3,1,1} \\ C_{3,1,2} \\ C_{3,1,3} \\ C_{3,1,4} \end{array} \begin{array}{c} C_{3,1,1} \quad C_{3,1,2} \quad C_{3,1,3} \quad C_{3,1,4} \\ \begin{pmatrix} 1 & 7 & 5 & 5 \\ 1/7 & 1 & 1/5 & 1/5 \\ 1/5 & 5 & 1 & 1 \\ 1/5 & 5 & 1 & 1 \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ C_{4,1,1} \\ C_{4,1,2} \\ C_{4,1,3} \\ C_{4,1,4} \end{array} \begin{array}{c} C_{4,1,1} \quad C_{4,1,2} \quad C_{4,1,3} \quad C_{4,1,4} \\ \begin{pmatrix} 1 & 5 & 3 & 3 \\ 1/5 & 1 & 1/3 & 1/3 \\ 1/3 & 3 & 1 & 1 \\ 1/3 & 3 & 1 & 1 \end{pmatrix} \end{array}$$

Comparision matrix for the first dimension

We calculate the weights of the dimensions from the formula 8, eventually we find the following weights:

Table 9. Relative weights of the criteria of the first dimension

D_1				D_2			
$P_{1,1,1}$	$P_{1,1,2}$	$P_{1,1,3}$	$P_{1,1,4}$	$P_{2,1,1}$	$P_{2,1,2}$	$P_{2,1,3}$	$P_{3,1,4}$
0.52	0.08	0.2	0.2	0.52	0.08	0.2	0.2
D_3				D_4			
$P_{3,1,1}$	$P_{3,1,2}$	$P_{3,1,3}$	$P_{3,1,4}$	$P_{4,1,1}$	$P_{4,1,2}$	$P_{4,1,3}$	$P_{4,1,4}$
0.61	0.05	0.17	0.17	0.52	0.08	0.2	0.2

These are the relative weights of the criteria (capacity, frequency, commercial speed, punctuality) of the first dimension, performance and services rendered. They represent the importance of a criterion with respect to the other in the same dimension in terms of each maker.

The four makers to agree on the order of importance of the selected criteria. For them the capacity of medium of transport is the most important, the second level of importance there is the commercial speed and punctuality, the less important is the frequency. Yet they are given the same weight for the four criteria.

We can also calculate the global weight of the criteria defined by 9 from the relative weights of the criteria and weight dimensions.

Table 10. Global weights of the criteria of the first dimension

	D_1				D_2			
Weight dimensions	$P_{1,1} = 0.65$				$P_{2,1} = 0.75$			
Relative weight	$P_{1,1,1}$	$P_{1,1,2}$	$P_{1,1,3}$	$P_{1,1,4}$	$P_{2,1,1}$	$P_{2,1,2}$	$P_{2,1,3}$	$P_{3,1,4}$
	0.52	0.08	0.2	0.2	0.52	0.08	0.2	0.2
Global weight	$w_{1,1,1}$	$w_{1,1,2}$	$w_{1,1,3}$	$w_{1,1,4}$	$w_{2,1,1}$	$w_{2,1,2}$	$w_{2,1,3}$	$w_{3,1,4}$
	0.338	0.052	0.13	0.13	0.39	0.06	0.15	0.15

	D_3				D_4			
Weight dimensions	$P_{3,1} = 0.28$				$P_{4,1} = 0.6$			
Relative weight	$P_{3,1,1}$	$P_{3,1,2}$	$P_{3,1,3}$	$P_{3,1,4}$	$P_{4,1,1}$	$P_{4,1,2}$	$P_{4,1,3}$	$P_{4,1,4}$
	0.61	0.05	0.17	0.17	0.52	0.08	0.2	0.2
Global weight	$w_{3,1,1}$	$w_{3,1,2}$	$w_{3,1,3}$	$w_{3,1,4}$	$w_{4,1,1}$	$w_{4,1,2}$	$w_{4,1,3}$	$w_{4,1,4}$
	0.52	0.08	0.2	0.2	0.52	0.08	0.2	0.2

The second dimension (cost) contains a single criterion, and the third dimension (environment). The following table lists the different weights associated with these two dimensions.

Because the second "cost" dimension contains a single criterion for all decision-makers while its relative weight $P_{i,2,1} = 1$ for i = 1,2,3,4. We compare with itself, therefore its global weight $w_{i,2,1}$ is by definition equal to

$$w_{3,2,1} = P_{i,2} \times P_{i,2,1} \quad \forall\, i = 1, .., 4$$

The same reasoning for the third dimension "environment".

Table 11. Weights of 2[sd] and 3[th] dimensions

	D_1		D_2	
Weight dimensions	$P_{1,2} = 0.28$	$P_{1,3} = 0.07$	$P_{2,2} = 0.2$	$P_{2,3} = 0.05$
Relative weight	$P_{1,2,1} = 1$	$P_{1,3,1} = 1$	$P_{2,2,1} = 1$	$P_{2,3,1} = 1$
Global weight	$w_{1,2,1} = 0.28$	$w_{1,3,1} = 0.07$	$w_{2,2,1} = 0.2$	$w_{2,3,1} = 0.05$
	D_3		D_4	
Weight dimensions	$P_{3,2} = 0.65$	$P_{3,3} = 0.07$	$P_{4,2} = 0.2$	$P_{4,3} = 0.2$
Relative weight	$P_{3,2,1} = 1$	$P_{3,3,1} = 1$	$P_{4,2,1} = 1$	$P_{4,3,1} = 1$
Global weight	$w_{3,2,1} = 0.65$	$w_{3,3,1} = 0.07$	$w_{4,2,1} = 0.2$	$w_{4,3,1} = 0.2$

6.4 Performance criteria

As we explained in the weighted sum method to minimize, the criteria should be changed so that each criterion is to maximize. In our problem, we have:

- Capacity: This is a criterion to be maximized.
- Frequency: This is a criterion to be minimized.
- Commercial speed: This is a criterion to be maximized.
- Punctuality: It is a criterion to be maximized.
- Investment cost: It is a criterion to be minimized.
- GES: It is a criterion to be minimized.

Each criterion is characterized by an evaluation function to transform a qualitative concept to a measurable quantitative concept. Now we are in this application based on real data from the Tramway Rabat Sale company without knowing the writing of evaluation functions. The following table includes all ratings.

Table 12. Real performances

	$C_{i,1,1}$	$C_{i,1,2}$	$C_{i,1,3}$	$C_{i,1,4}$	$C_{i,2,1}$	$C_{i,3,1}$
Tramway a_1	560	9 min	18.25 Km/h	97 %	3814 MDH	0
Bus a_2	175	20 min	12 Km/h	70 %	2 MDH	480 gCO_2

The equivalence relation 6 serves to transform the objective of minimizing a maximization objective. A help of this relationship we transform performance values of this last table with the following table:

Table 13. Performance after the transformation

	$C_{i,1,1}$	$C_{i,1,2}$	$C_{i,1,3}$	$C_{i,1,4}$	$C_{i,2,1}$	$C_{i,3,1}$
Tramway a_1	560	11	18.25	97	0	480
Bus a_2	175	0	12	70	0.814	0

To agglomerate these criteria into a single score, a normalization step allows to transpose the different units on the same comparable scale. There are various formulas standardization, for translating the various units on a scale of 0 to 1. One that has kept us in this model is the simplest. Indeed we divisions each performance value on the sum of the column which normalizes the values, so we find the following table:

Table 14. The normalized performance

	$C_{i,1,1}$	$C_{i,1,2}$	$C_{i,1,3}$	$C_{i,1,4}$	$C_{i,2,1}$	$C_{i,3,1}$
Tramway a_1	0.76	1	0.603	0.584	0	1
Bus a_2	0.238	0	0.396	0.419	1	0

In this step, we have specified all dimensions, their weight, the set of criteria, their weight and their evaluations, which calculates the weighted sums for using the formula 11. These are the same criteria considered for the four-makers.

Moreover, makers D_1, D_2 and D_4 have chosen the same weight criteria for the performance and rendered services dimension, so we come to the same table for weighted sums for

i = 1, 2, 4.

Table 15. Relative weighted sum of D_1, D_2 and D_4

Dimension $d_{i,1}$ i=1,2,4						
Criterion		$C_{i,1,1}$	$C_{i,1,2}$	$C_{i,1,3}$	$C_{i,1,4}$	**Sum**
Relative weight		0.52	0.08	0.2	0.2	$\sum_{p=1}^{4} P_{i,1,p} = 1$
Alternative	a_1	0.76	1	0.603	0.584	${S_{i,1}}^m(a_1) = 0.7126$
	a_2	0.238	0	0.396	0.419	${S_{i,1}}^m(a_2) = 0.2867$

Table 16. Relative weighted sum of D_3

Dimension $d_{3,1}$						
Criterion		$C_{3,1,1}$	$C_{3,1,2}$	$C_{3,1,3}$	$C_{3,1,4}$	**Sum**
Relative weight		0.61	0.05	0.17	0.17	$\sum_{p=1}^{4} P_{3,1,p} = 1$
Alternative	a_1	0.76	1	0.603	0.584	${S_{3,1}}^m(a_1) = 0.71539$
	a_2	0.238	0	0.396	0.419	${S_{3,1}}^m(a_2) = 0.238373$

At the level of performance and rendered services we find that the tramway meets the needs of citizens better than the weight of the bus for the four-makers. Following the action Tramway (a_1) is the optimal action relative to the first dimensions.

If the aim of the study is to compare the two transport Tram and bus for the three chosen dimensions, so decision makers should calculate overall sums defined by the formula 6. The following tables summarize the results found by the four makers:

Table 17. Global weighted sums of D_1

Dimension		$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	**Overall sum**
Weight		0.65	0.28	0.07	
Alternative	a_1	${S_{1,1}}^m(a_1) = 0.7126$	0	1	$S_1^m(a_1) = 0.5328$
	a_2	${S_{1,1}}^m(a_2) = 0.2867$	1	0	$S_1^m(a_2) = 0.466$

Table 18. Global weighted sums of D_2

Dimension		$d_{2,1}$	$d_{2,2}$	$d_{2,3}$	**Overall sum**
Weight		0.75	0.2	0.005	
Alternative	a_1	${S_{2,1}}^m(a_1) = 0.7126$	0	1	$S_2^m(a_1) = 0.584$
	a_2	${S_{2,1}}^m(a_2) = 0.2867$	1	0	$S_2^m(a_2) = 0.4145$

Table 19. Global weighted sums of D_3

Dimension		$d_{3,1}$	$d_{3,2}$	$d_{3,3}$	**Overall sum**
Weight		0.28	0.65	0.07	
Alternative	a_1	${S_{3,1}}^m(a_1) = 0.71539$	0	1	$S_3^m(a_1) = 0.27$
	a_2	${S_{3,1}}^m(a_2) = 0.238373$	1	0	$S_3^m(a_2) = 0.71664$

Table 20. Global weighted sums of D_4

Dimension		$d_{4,1}$	$d_{4,2}$	$d_{4,3}$	Overall sum
Weight		0.6	0.2	0.2	
Alternative	a_1	${S_{4,1}}^m(a_1) = 0.7126$	0	1	$S_4^m(a_1) = 0.627$
	a_2	${S_{4,1}}^m(a_2) = 0.2867$	1	0	$S_4^m(a_2) = 0.3716$

For the four makers we always $S_i^m(a_1) > S_i^m(a_2)$ i= 1, 2, 4, so the tram is optimal action this modeling.

If the comparison is done just at the second dimension "investment costs" then the action a_2 "Bus" will be the best for the four-makers and this result is clearly seen that the project of Tram company Rabat Salle required an investment cost of 3.814 MDH or that of the bus asked two MDH.

7 Conclusion

The fact of transforming the multi-criteria multi-decision makers problems in multi-criteria mono-decision maker problems, allowed us to choose optimal solutions for all the decision-makers in a group but independently, according to the dimensions and the simultaneously considered criteria.

This work proposes a method to resolve the question of decision aid, obtained from the composition of the AHP method, which gives a scientific writing to the concept of weighting, and the weighted sums method that provides a single criterion in the form of a numeric result. Consequently, we arrived at mathematical expressions leaving to a programmable algorithm of ordonnement of the set of potential actions.

Our proposal is a mathematical modelling which is applicable for a large class of decision problems, moreover the factor of time introduced allows to study even dynamic appearance problems.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Alexander Melvin. Decision-making using the analytic hierarchy process (AHP) and SAS/IML. SESUG, Paper SD-04; 2012.

[2] Henreit L. Evaluation system and multi-criteria classification. Paris Dauphine University; 2000. French.

[3] Talbi E. Metaheuristics for multi-objective combinatorial optimization. Paris: Tutorial, Evolutionary Quarterly Day. Paris; 1999. French.

[4] Berkoune D. Evaluation approach for advanced scheduling problems: Aggregation method with dynamic search direction. SMC 276-281. French.

[5] Vincent C. Thesis: Systematic approach and multi-criteria approach to the definition of a system of performance indicators; 2004.

[6] Mammeri Mohamed Lounes. Thesis: A multicriteria approach to decision making for the evaluation of comfort in trains. Paris; 2013. French.

[7] Nafi A. Question multicriteria decision: Introduction to multi-criteria analysis methods Electra type. ENGEES Module: Financial Engineering; 2009-2010.

[8] Thomas L. Saaty. Decision making for Leders, published by: Lifetme Learning Publications, a division of Wadsworth, INC., 10 David Drive, Belmont, CA, 94002 USA.

[9] Fishbum Peter C. Additive utilities with incomplete product set: Applications to priorities and assignments. Operations Research Society of America (ORSA), Baltimore, MD, USA; 1967.
Available: https://cours.etsmtl.ca/ing802/matiere/pdf
www.cirano.qc.ca/pdf/publication/2003s-53.pdf
eprints.lse.ac.uk/12761/1/Multi-criteria_Analysis.pdf

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://sciencedomain.org/review-history/13197