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Global Attractivity and Almost Periodic Solution of a Discrete Multispecies Lotka-Volterra Mutualism System with Feedback Controls

Hui Zhang[1*]

[1] *Mathematics and OR Section, Xi'an Research Institute of High-tech Hongqing Town, Xi'an, Shaanxi 710025, China.*

Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

In this paper, we consider an almost periodic discrete multispecies Lotka-Volterra mutualism system with feedback controls. We firstly obtain the permanence of the system. Assuming that the coefficients in the system are almost periodic sequences, we obtain the sufficient conditions for the existence of a unique almost periodic solution which is globally attractive. An example together with numerical simulation indicates the feasibility of the main result.

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Corresponding author: E-mail: zh53054958@163.com;

1 Introduction

In 2015, Zhang et al.[1] had studied the following discrete multispecies Lotka-Volterra mutualism system with feedback controls

$$\begin{cases} x_i(k+1) = x_i(k)\exp\left\{a_i(k) - b_i(k)x_i(k) + \sum\limits_{j=1,j\neq i}^{n} c_{ij}(k)\dfrac{x_j(k)}{d_{ij}(k)+x_j(k)} - e_i(k)u_i(k)\right\}, \\ \Delta u_i(k) = -f_i(k)u_i(k) + \sum\limits_{j=1}^{n} g_{ij}(k)x_j(k), \quad i = 1, 2, \cdots, n, \end{cases} \tag{1.1}$$

where $\{a_i(k)\}, \{b_i(k)\}, \{c_{ij}(k)\}, \{d_{ij}(k)\}, \{e_i(k)\}, \{f_i(k)\}$ and $\{g_{ij}(k)\}$ are bounded nonnegative almost periodic sequences such that

$$\begin{aligned} &0 < a_i^l \le a_i(k) \le a_i^u,\ 0 < b_i^l \le b_i(k) \le b_i^u, 0 < c_{ij}^l \le c_{ij}(k) \le c_{ij}^u,\ 0 < d_{ij}^l \le d_{ij}(k) \le d_{ij}^u, \\ &0 < e_i^l \le e_i(k) \le e_i^u,\ 0 < f_i^l \le f_i(k) \le f_i^u < 1,\ 0 < g_{ij}^l \le g_{ij}(k) \le g_{ij}^u, \end{aligned} \tag{1.2}$$

$i, j = 1, 2, \cdots, n,\ k \in \mathbf{Z}$. By means of constructing a suitable Lyapunov function, sufficient conditions are obtained for the existence of a unique positive almost periodic solution which is uniformly asymptotically stable.

For any bounded sequence $\{f(k)\}$ defined on $\mathbf{Z}$, $f^u = \sup\limits_{k\in\mathbf{Z}} f(k), f^l = \inf\limits_{k\in\mathbf{Z}} f(k)$. By the biological meaning, we will focus our discussion on the positive solutions of system (1.1). So it is assumed that the initial conditions of system (1.1) are the form:

$$x_i(0) > 0, u_i(0) > 0,\ i = 1, 2, \cdots, n. \tag{1.3}$$

One can easily show that the solutions of system (1.1) with the initial condition (1.3) are defined and remain positive for all $k \in N^+ = \{0, 1, 2, 3, \cdots\}$.

With the stimulation from the works[2, 3, 4, 5], the main purpose of this paper is to obtain a set of sufficient conditions to ensure the existence of a unique globally attractive positive almost periodic solution of system (1.1) with initial condition (1.3).

The remaining part of this paper is organized as follows: In Section 2, we will introduce some definitions and several useful lemmas. In the next section, we establish the permanence of system (1.1). Then, in Section 4, we establish sufficient conditions to ensure the existence of a unique positive almost periodic solution which is globally attractive. The main results are illustrated by an example with numerical simulation in Section 5. Finally, the conclusion ends with brief remarks in the last section.

2 Preliminaries

First, we give the definitions of the terminologies involved.

Definition 2.1([6, 7]) A sequence $x : Z \to R$ is called an almost periodic sequence if the ε-translation set of x

$$E\{\varepsilon, x\} = \{\tau \in Z : \mid x(n+\tau) - x(n) \mid < \varepsilon, \forall n \in Z\}$$

is a relatively dense set in Z for all $\varepsilon > 0$; that is, for any given $\varepsilon > 0$, there exists an integer $l(\varepsilon) > 0$ such that each interval of length $l(\varepsilon)$ contains an integer $\tau \in E\{\varepsilon, x\}$ with

$$\mid x(n+\tau) - x(n) \mid < \varepsilon, \quad \forall n \in Z.$$

τ is called an ε-translation number of $x(n)$.

Definition 2.2([8]) Let D be an open subset of R^m, $f : Z \times D \to R^m$. $f(n, x)$ is said to be almost periodic in n uniformly for $x \in D$ if for any $\varepsilon > 0$ and any compact set $S \subset D$, there exists a positive integer $l = l(\varepsilon, S)$ such that any interval of length l contains an integer τ for which

$$|f(n+\tau, x) - f(n, x)| < \varepsilon, \quad \forall (n, x) \in Z \times S.$$

τ is called an ε-translation number of $f(n, x)$.

Definition 2.3([9]) A sequence $x : Z^+ \to R$ is called an asymptotically almost periodic sequence if

$$x(n) = p(n) + q(n), \quad \forall n \in Z^+,$$

where $p(n)$ is an almost periodic sequence and $\lim\limits_{n\to+\infty} q(n) = 0$.

Definition 2.4 A solution $(x_1(k), x_2(k), \cdots, x_n(k), u_1(k), u_2(k), \cdots, u_n(k))$ of system (1.1) with initial condition (1.3) is said to be globally attractive if for any other solution $(x_1^*(k), x_2^*(k), \cdots, x_n^*(k), u_1^*(k), u_2^*(k), \cdots, u_n^*(k))$ of system (1.1) with initial condition (1.3), we have

$$\lim_{k\to+\infty} (x_i^*(k) - x_i(k)) = 0, \qquad \lim_{k\to+\infty} (u_i^*(k) - u_i(k)) = 0, \quad i = 1, 2, \cdots, n.$$

Now, we state several lemmas which will be useful in proving our main result.

Lemma 2.1([10]) $\{x(n)\}$ is an almost periodic sequence if and only if for any integer sequence $\{k_i'\}$, there exists a subsequence $\{k_i\} \subset \{k_i'\}$ such that the sequence $\{x(n+k_i)\}$ converges uniformly for all $n \in Z$ as $i \to \infty$. Furthermore, the limit sequence is also an almost periodic sequence.

Lemma 2.2([11]) $\{x(n)\}$ is an asymptotically almost periodic sequence if and only if, for any sequence $m_i \subset Z$ satisfying $m_i > 0$ and $m_i \to \infty$ as $i \to \infty$ there exists a subsequence $\{m_{i_k}\} \subset \{m_i\}$ such that the sequence $\{x(n + m_{i_k})\}$ converges uniformly for all $n \in Z^+$ as $k \to \infty$.

Lemma 2.3([12]) Assume that $\{x(n)\}$ satisfies $x(n) > 0$ and

$$x(n+1) \le x(n) \exp\{a(n) - b(n)x(n)\}$$

for $n \in \mathbf{N}$, where $a(n)$ and $b(n)$ are non-negative sequences bounded above and below by positive constants. Then

$$\limsup_{n\to+\infty} x(n) \le \frac{1}{b^l} \exp\{a^u - 1\}.$$

Lemma 2.4([12]) Assume that $\{x(n)\}$ satisfies

$$x(n+1) \ge x(n) \exp\{a(n) - b(n)x(n)\}, \quad n \ge N_0,$$

$$\limsup_{n\to+\infty} x(n) \le x^*,$$

and $x(N_0) > 0$, where $a(n)$ and $b(n)$ are non-negative sequences bounded above and below by positive constants and $N_0 \in N$. Then

$$\liminf_{n\to+\infty} x(n) \ge \min\left\{\frac{a^l}{b^u} \exp\{a^l - b^u x^*\}, \frac{a^l}{b^u}\right\}.$$

Lemma 2.5([13]) Assume that $A > 0$ and $y(0) > 1$, and further suppose that

$$y(n+1) \le Ay(n) + B(n), \quad n = 1, 2, 3, \cdots.$$

Then for any integer $k \leq n$,

$$y(n) \leq A^k y(n-k) + \sum_{i=0}^{k-1} A^i B(n-i-1).$$

Especially, if $A < 1$ and B is bounded above with respect to M, then

$$\limsup_{n\to\infty} y(n) \leq \frac{M}{1-A}.$$

Lemma 2.6([13]) Assume that $A > 0$ and $y(0) > 1$, and further suppose that

$$y(n+1) \geq Ay(n) + B(n), \quad n = 1, 2, 3, \cdots.$$

Then for any integer $k \leq n$,

$$y(n) \geq A^k y(n-k) + \sum_{i=0}^{k-1} A^i B(n-i-1).$$

Especially, if $A < 1$ and B is bounded below with respect to m, then

$$\liminf_{n\to\infty} y(n) \geq \frac{m}{1-A}.$$

3 Permanence

In this section, we establish the permanence result for system (1.1). The proofs of following results can be found in [1] and we omit the details here.

Theorem 3.1([1]) Assume that the conditions (1.2) and (1.3) hold, furthermore,

$$a_i^l - e_i^u N_i > 0, \tag{3.1}$$

then system (1.1) is permanent, that is, there exist positive constants m_i, M_i, n_i and $N_i(i = 1, 2, \cdots, n)$ which are independent of the solutions of system (1.1), such that for any positive solution $(x_1(k), x_2(k), \cdots, x_n(k), u_1(k),\, u_2(k), \cdots, u_n(k))$ of system (1.1), one has:

$$\begin{aligned} &m_i \leq \liminf_{k\to+\infty} x_i(k) \leq \limsup_{k\to+\infty} x_i(k) \leq M_i, \\ &n_i \leq \liminf_{k\to+\infty} u_i(k) \leq \limsup_{k\to+\infty} u_i(k) \leq N_i, \qquad i = 1, 2, \cdots, n, \end{aligned}$$

where

$$M_i = \frac{1}{b_i^l} \exp\left\{a_i^u + \sum_{j=1, j\neq i}^{n} c_{ij}^u - 1\right\}, \quad m_i = \frac{a_i^l - e_i^u N_i}{2b_i^u} \min\left\{1, \exp\left\{a_i^l - e_i^u N_i - b_i^u M_i\right\}\right\}$$

$$N_i = \frac{1}{f_i^l} \sum_{j=1}^{n} g_{ij}^u M_j, \quad n_i = \frac{1}{f_i^u} \sum_{j=1}^{n} g_{ij}^l m_j.$$

We denote by Ω the set of all solutions $(x_1(k), x_2(k), \cdots, x_n(k), u_1(k), u_2(k), \cdots, u_n(k))$ of system (1.1) satisfying $m_i \leq x_i(k) \leq M_i, n_i \leq u_i(k) \leq N_i (i = 1, 2, \cdots, n)$ for all $k \in Z^+$.

Proposition 3.1([1]) Assume that the conditions (1.2), (1.3) and (3.1) hold. Then $\Omega \neq \Phi$.

4 Global Attractivity and Almost Periodic Solution

The main results of this paper concern the global attractivity of almost periodic solution of system (1.1) with conditions (1.2), (1.3) and (3.1).

Theorem 4.1 Assume that (1.2), (1.3), (3.1) and

$$\text{(H1)}\quad \rho_i = \max\{|1-\theta_{ii}b_i^l m_i^{\theta_{ii}}|,\ |1-\theta_{ii}b_i^u M_i^{\theta_{ii}}|\} + \sum_{j=1,j\neq i}^{n} \frac{\theta_{ij}c_{ij}^u M_j^{\theta_{ij}}}{d_{ij}^l} + e_i^u < 1,$$

$$\varphi_i = 1 - f_i^l + \sum_{j=1}^{n} g_{ij}^u M_j < 1,\ i=1,2,\cdots,n,$$

hold. Then any positive solution $(x_1(k), x_2(k), \cdots, x_n(k), u_1(k), u_2(k), \cdots, u_n(k))$ of system (1.1) is globally attractive.

Proof. Assume that $(p_1(k), p_2(k), \cdots, p_n(k), v_1(k), v_2(k), \cdots, v_n(k))$ is a solution of system (1.1) satisfying (1.2) and (1.3). Let

$$x_i(k) = p_i(k)\exp\{q_i(k)\},\quad u_i(k) = v_i(k) + w_i(k),\quad i=1,2,\cdots,n.$$

Since

$$\begin{aligned}
q_i(k+1) &= \ln x_i(k+1) - \ln p_i(k+1) \\
&= \ln x_i(k) + a_i(k) - b_i(k)(x_i(k))^{\theta_{ii}} + \sum_{j=1,j\neq i}^{n} c_{ij}(k)\frac{(x_j(k))^{\theta_{ij}}}{d_{ij}(k)+(x_j(k))^{\theta_{ij}}} - e_i(k)u_i(k) \\
&\quad - \ln p_i(k) - a_i(k) + b_i(k)(p_i(k))^{\theta_{ii}} - \sum_{j=1,j\neq i}^{n} c_{ij}(k)\frac{(p_j(k))^{\theta_{ij}}}{d_{ij}(k)+(p_j(k))^{\theta_{ij}}} + e_i(k)v_i(k) \\
&= q_i(k) - b_i(k)[(x_i(k))^{\theta_{ii}} - (p_i(k))^{\theta_{ii}}] \\
&\quad + \sum_{j=1,j\neq i}^{n} \frac{d_{ij}(k)c_{ij}(k)[(x_j(k))^{\theta_{ij}} - (p_j(k))^{\theta_{ij}}]}{[d_{ij}(k)+(x_j(k))^{\theta_{ij}}][d_{ij}(k)+(p_j(k))^{\theta_{ij}}]} - e_i(k)w_i(k) \\
&= q_i(k) - b_i(k)(p_i(k))^{\theta_{ii}}\left[(\exp\{q_i(k)\})^{\theta_{ii}} - 1\right] \\
&\quad + \sum_{j=1,j\neq i}^{n} \frac{d_{ij}(k)c_{ij}(k)(p_j(k))^{\theta_{ij}}[(\exp\{q_j(k)\})^{\theta_{ij}} - 1]}{[d_{ij}(k)+(x_j(k))^{\theta_{ij}}][d_{ij}(k)+(p_j(k))^{\theta_{ij}}]} - e_i(k)w_i(k) \\
&= q_i(k)\big(1 - \theta_{ii}b_i(k)[p_i(k)\exp\{\lambda_i(k)q_i(k)\}]^{\theta_{ii}}\big) \\
&\quad + \sum_{j=1,j\neq i}^{n} \frac{d_{ij}(k)\theta_{ij}c_{ij}(k)q_j(k)[p_j(k)\exp\{\overline{\lambda_j}(k)q_j(k)\}]^{\theta_{ij}}}{[d_{ij}(k)+(x_j(k))^{\theta_{ij}}][d_{ij}(k)+(p_j(k))^{\theta_{ij}}]} - e_i(k)w_i(k),
\end{aligned} \tag{4.1}$$

where $\lambda_i(k), \overline{\lambda_j}(k) \in (0,1)$.

Similarly, we get

$$\begin{aligned} w_i(k+1) &= u_i(k+1) - v_i(k+1) \\ &= (1-f_i(k))u_i(k) + \sum_{j=1}^{n} g_{ij}(k)x_j(k) - (1-f_i(k))v_i(k) - \sum_{j=1}^{n} g_{ij}(k)p_j(k) \\ &= (1-f_i(k))w_i(k) + \sum_{j=1}^{n} g_{ij}(k)p_j(k)(\exp\{q_j(k)\}-1) \\ &= (1-f_i(k))w_i(k) + \sum_{j=1}^{n} g_{ij}(k)p_j(k)q_j(k)\exp\{\xi_j(k)q_j(k)\}, \quad i=1,2,\cdots,n, \end{aligned}$$

where $\xi_j(k) \in (0,1)$.

To complete the proof, it suffices to show that

$$\lim_{k\to+\infty} q_i(k) = 0, \lim_{k\to+\infty} w_i(k) = 0, \quad i=1,2,\cdots,n. \tag{4.2}$$

In view of (H2), we can choose $\varepsilon > 0$ such that

$$\rho_i^{\varepsilon} = \max\{|1-\theta_{ii}b_i^l(m_i-\varepsilon)^{\theta_{ii}}|,\ |1-\theta_{ii}b_i^u(M_i+\varepsilon)^{\theta_{ii}}|\} + \sum_{j=1,j\neq i}^{n} \frac{\theta_{ij}c_{ij}^u(M_j+\varepsilon)^{\theta_{ij}}}{d_{ij}^l} + e_i^u < 1,$$

$$\varphi_i^{\varepsilon} = 1 - f_i^l + \sum_{j=1}^{n} g_{ij}^u(M_j+\varepsilon) < 1, \qquad i=1,2,\cdots,n.$$

Let $\rho = \max\{\rho_i^{\varepsilon}, \varphi_i^{\varepsilon}\}$, then $\rho < 1$. According to Theorem 3.2, there exists a positive integer $k_0 \in Z^+$ such that

$$m_i - \varepsilon \le x_i(k) \le M_i + \varepsilon, \quad m_i - \varepsilon \le p_i(k) \le M_i + \varepsilon, \quad i=1,2,\cdots,n$$

for $k \ge k_0$.

Notice that $\lambda_i(k) \in [0,1]$ implies that $p_i(k)\exp\{\lambda_i(k)u_i(k)\}$ lies between $p_i(k)$ and $x_i(k)$, $\overline{\lambda_j}(k) \in [0,1]$ implies that $p_j(k)\exp\{\overline{\lambda_j}(k)u_j(k)\}$ lies between $p_j(k)$ and $x_j(k)$. From (4.1), we get

$$\begin{aligned} |q_i(k+1)| &\le \max\{|1-\theta_{ii}b_i^l(m_i-\varepsilon)^{\theta_{ii}}|,\ |1-\theta_{ii}b_i^u(M_i+\varepsilon)^{\theta_{ii}}|\}|q_i(k)| \\ &\quad + \sum_{j=1,j\neq i}^{n} \frac{\theta_{ij}c_{ij}^u(M_j+\varepsilon)^{\theta_{ij}}}{d_{ij}^l}|q_j(k)| + e_i^u|w_i(k)|, \\ |w_i(k+1)| &\le (1-f_i^l)|w_i(k)| + \sum_{j=1}^{n} g_{ij}^u(M_j+\varepsilon)|q_j(k)|, \quad i=1,2,\cdots,n, \end{aligned} \tag{4.3}$$

for $k \ge k_0$.

In view of (4.3), we get

$$\max\{\max_{1\le i\le n}|q_i(k+1)|, \max_{1\le i\le n}|w_i(k+1)|\} \le \rho\max\{\max_{1\le i\le n}|q_i(k)|, \max_{1\le i\le n}|w_i(k)|\}, \quad k \ge k_0.$$

This implies

$$\max\{\max_{1\le i\le n}|q_i(k)|, \max_{1\le i\le n}|w_i(k)|\} \le \rho^{k-k_0}\max\{\max_{1\le i\le n}|q_i(k_0)|, \max_{1\le i\le n}|w_i(k_0)|\}, \quad k \ge k_0.$$

Then (4.2) holds and we can obtain

$$\lim_{k\to+\infty}|x_i(k)-p_i(k)| = 0, \quad \lim_{k\to+\infty}|u_i(k)-v_i(k)| = 0, \quad i=1,2,\cdots,n. \tag{4.4}$$

Therefore, positive solution $(x_1(k), x_2(k), \cdots, x_n(k), u_1(k), u_2(k), \cdots, u_n(k))$ of system (1.1) is globally attractive. $\square$

Theorem 4.2 Assume that (1.2), (1.3), (3.1) and (H1) hold. Then system (1.1) admits a unique almost periodic solution which is globally attractive.

Proof. It follows from Proposition 3.1 that there exists a solution $(x_1(k), x_2(k), \cdots, x_n(k), u_1(k), u_2(k), \cdots, u_n(k))$ of system (1.1) satisfying $m_i \le x_i(k) \le M_i, n_i \le u_i(k) \le N_i, k \in Z^+$.

Suppose that $(x_1(k), x_2(k), \cdots, x_n(k), u_1(k), u_2(k), \cdots, u_n(k))$ is any solution of system (1.1), then there exists an integer valued sequence $\{k_p^{'}\}$, $k_p^{'} \to +\infty$ as $p \to +\infty$, such that $(x_1(k+k_p^{'}), x_2(k+k_p^{'}), \cdots, x_n(k+k_p^{'}), u_1(k+k_p^{'}), u_2(k+k_p^{'}), \cdots, u_n(k+k_p^{'}))$ is a solution of the following system

$$\begin{cases} x_i(k+1) = x_i(k)\exp\Big\{a_i(k+k_p^{'}) - b_i(k+k_p^{'})x_i(k) + \displaystyle\sum_{j=1, j\neq i}^{n} c_{ij}(k+k_p^{'})\frac{x_j(k)}{d_{ij}(k+k_p^{'}) + x_j(k)} \\ \qquad\qquad -e_i(k+k_p^{'})u_i(k)\Big\}, \\ \Delta u_i(k) = -f_i(k+k_p^{'})u_i(k) + \displaystyle\sum_{j=1}^{n} g_{ij}(k+k_p^{'})x_j(k), \quad i = 1, 2, \cdots, n, \end{cases}$$

From above discussion and Theorem 3.1, we have that not only $(x_1(k+k_p^{'}), x_2(k+k_p^{'}), \cdots, x_n(k+k_p^{'}), u_1(k+k_p^{'}), u_2(k+k_p^{'}), \cdots, u_n(k+k_p^{'}))$ but also $(\Delta x_1(k+k_p^{'}), \Delta x_2(k+k_p^{'}), \cdots, \Delta x_n(k+k_p^{'}), \Delta u_1(k+k_p^{'}), \Delta u_2(k+k_p^{'}), \cdots, \Delta u_n(k+k_p^{'}))$ are uniformly bounded, thus $(x_1(k+k_p^{'}), x_2(k+k_p^{'}), \cdots, x_n(k+k_p^{'}), u_1(k+k_p^{'}), u_2(k+k_p^{'}), \cdots, u_n(k+k_p^{'}))$ are uniformly bounded and equi-continuous. By Ascoli's theorem[14], there exists a uniformly convergent subsequence $(x_1(k+k_p), x_2(k+k_p), \cdots, x_n(k+k_p), u_1(k+k_p), u_2(k+k_p), \cdots, u_n(k+k_p)) \subseteq (x_1(k+k_p^{'}), x_2(k+k_p^{'}), \cdots, x_n(k+k_p^{'}), u_1(k+k_p^{'}), u_2(k+k_p^{'}), \cdots, u_n(k+k_p^{'}))$ such that for any $\varepsilon > 0$, there exists a $k_0(\varepsilon) > 0$ with the property that if $m, n \ge k_0(\varepsilon)$ then

$$|x_i(k+k_m) - x_i(k+k_n)| < \varepsilon, \quad |u_i(k+k_m) - u_i(k+k_n)| < \varepsilon,$$

which shows from Lemma 2.2 that $(x_1(k+k_n), x_2(k+k_n), \cdots, x_n(k+k_n), u_1(k+k_n), u_2(k+k_n), \cdots, u_n(k+k_n))$ is asymptotically almost periodic sequence, then $(x_1(k+k_n), x_2(k+k_n), \cdots, x_n(k+k_n), u_1(k+k_n), u_2(k+k_n), \cdots, u_n(k+k_n))$ are the sum of an almost periodic sequence $(p_1(k+k_n), p_2(k+k_n), \cdots, p_n(k+k_n), v_1(k+k_n), v_2(k+k_n), \cdots, v_n(k+k_n))$ and a sequence $(q_1(k+k_n), q_2(k+k_n), \cdots, q_n(k+k_n), w_1(k+k_n), w_2(k+k_n), \cdots, w_n(k+k_n))$ defined on Z, such that

$$x_i(k+k_n) = p_i(k+k_n) + q_i(k+k_n), u_i(k+k_n) = v_i(k+k_n) + w_i(k+k_n), \quad k \in Z,$$

where

$$\lim_{n\to+\infty} p_i(k+k_n) = p_i(k), \quad \lim_{n\to+\infty} v_i(k+k_n) = v_i(k),$$
$$\lim_{n\to+\infty} q_i(k+k_n) = 0, \quad \lim_{n\to+\infty} w_i(k+k_n) = 0,$$

$\{p_i(k)\}$ and $\{v_i(k)\}$ are almost periodic sequences, $i = 1, 2, \cdots, n$. It means that

$$\lim_{n\to+\infty} x_i(k+k_n) = p_i(k), \quad \lim_{n\to+\infty} u_i(k+k_n) = v_i(k).$$

In the following we show that $\{(p_1(k), p_2(k), \cdots, p_n(k), v_1(k), v_2(k), \cdots, v_n(k))\}$ is an almost periodic solution of system (1.1).

From the properties of an almost periodic sequence, there exists an integer valued sequence $\{\delta_p\}$,

$\delta_p \to +\infty$ as $p \to +\infty$, such that

$$a_i(k+\delta_p) \to a_i(k), \quad b_i(k+\delta_p) \to b_i(k), \quad c_{ij}(k+\delta_p) \to c_{ij}(k), \quad d_{ij}(k+\delta_p) \to d_{ij}(k),$$
$$e_i(k+\delta_p) \to e_i(k), \quad f_i(k+\delta_p) \to f_i(k), \quad g_{ij}(k+\delta_p) \to g_{ij}(k), \quad \text{as} \quad p \to +\infty.$$

It is easy to know that $x_i(k+\delta_p) \to p_i(k)$, $u_i(k+\delta_p) \to v_i(k)$ as $p \to \infty$, then we have

$$\begin{aligned} p_i(k+1) &= \lim_{p\to\infty} x_i(k+1+\delta_p) \\ &= \lim_{p\to\infty} x_i(k+\delta_p) \exp\Big\{ a_i(k+\delta_p) - b_i(k+\delta_p)x_i(k+\delta_p) \\ &\quad + \sum_{j=1,j\neq i}^{n} c_{ij}(k+\delta_p)\frac{x_j(k+\delta_p)}{d_{ij}(k+\delta_p)+x_j(k+\delta_p)} - e_i(k+\delta_p)u_i(k+\delta_p)\Big\} \\ &= p_i(k)\exp\Big\{ a_i(k) - b_i(k)p_i(k) + \sum_{j=1,j\neq i}^{n} c_{ij}(k)\frac{p_j(k)}{d_{ij}(k)+p_j(k)} - e_i(k)v_i(k)\Big\}, \end{aligned}$$

$$\begin{aligned} v_i(k+1) &= \lim_{p\to\infty} u_i(k+1+\delta_p) \\ &= \lim_{p\to\infty} \Big\{ [1-f_i(k+\delta_p)]u_i(k+\delta_p) + \sum_{j=1}^{n} g_{ij}(k+\delta_p)x_j(k+\delta_p)\Big\} \\ &= [1-f_i(k)]v_i(k) + \sum_{j=1}^{n} g_{ij}(k)p_j(k). \end{aligned}$$

This prove that $p(k) = \{(p_1(k), p_2(k), \cdots, p_n(k), v_1(k), v_2(k), \cdots, v_n(k))\}$ satisfied system (1.1), and $p(k)$ is a positive almost periodic solution of system (1.1).

Now, we show that there is only one positive almost periodic solution of system (1.1). For any two positive almost periodic solutions $(p_1(k), p_2(k), \cdots, p_n(k), v_1(k), v_2(k), \cdots, v_n(k))$ and $(z_1(k), z_2(k), \cdots, z_n(k), l_1(k), l_2(k), \cdots, l_n(k))$ of system (1.1), we claim that $p_i(k) = z_i(k), v_i(k) = l_i(k), (i = 1, 2, \cdots, n)$ for all $k \in \mathbf{Z}^+$. Otherwise there must be at least one positive integer $K^* \in \mathbf{Z}^+$ such that $p_i(K^*) \neq z_i(K^*)$ or $v_j(K^*) \neq l_j(K^*)$ for a certain positive integer i or j, i.e., $\Omega_1 = |p_i(K^*) - z_i(K^*)| > 0$ or $\Omega_2 = |v_j(K^*) - l_j(K^*)| > 0$. So we can easily know that

$$\begin{aligned} \Omega_1 = |\lim_{p\to+\infty} p_i(K^*+\delta_p) - \lim_{p\to+\infty} z_i(K^*+\delta_p)| &= \lim_{p\to+\infty} |p_i(K^*+\delta_p) - z_i(K^*+\delta_p)| \\ &= \lim_{k\to+\infty} |p_i(k) - z_i(k)| > 0, \end{aligned}$$

or

$$\begin{aligned} \Omega_2 = |\lim_{p\to+\infty} v_j(K^*+\delta_p) - \lim_{p\to+\infty} l_j(K^*+\delta_p)| &= \lim_{p\to+\infty} |v_j(K^*+\delta_p) - l_j(K^*+\delta_p)| \\ &= \lim_{k\to+\infty} |v_j(k) - l_j(k)| > 0, \end{aligned}$$

which is a contradiction to (4.4). Thus $p_i(k) = z_i(k), v_i(k) = l_i(k) (i = 1, 2, \cdots, n)$ hold for $\forall k \in \mathbf{Z}^+$. Therefore, system (1.1) admits a unique almost periodic solution which is globally attractive. This completes the proof of Theorem 4.2. □

Remark 4.1 If $n = 2$, the conditions of Theorem 4.2 can be simplified. Therefore, we have the following result.

Corollary 4.1 Let $n = 2$, and assume further that (1.2), (1.3), (3.1) and

$$\text{(H2)}\quad \rho_i = \max\{|1-\theta_{ii}b_i^l m_i^{\theta_{ii}}|,\ |1-\theta_{ii}b_i^u M_i^{\theta_{ii}}|\} + \frac{\theta_{ij}c_{ij}^u M_j^{\theta_{ij}}}{d_{ij}^l} + e_i^u < 1,$$
$$\varphi_i = 1 - f_i^l + g_{ii}^u M_i + g_{ij}^u M_j < 1,\ i,j = 1,2, i \neq j,$$

hold. Then system (1.1) admits a unique globally attractive almost periodic solution $(x_1(k), x_2(k), u_1(k), u_2(k))$ which is bounded by Ω for all $k \in Z^+$.

5 Example and Numerical Simulation

In this section, we give the following example to check the feasibility of our result.

Example Consider the following almost periodic discrete Lotka-Volterra mutualism system with feedback controls

$$\begin{aligned}
x_1(k+1) &= x_1(k)\exp\Big\{1.1 - 0.022\sin(\sqrt{3}k) - (1.05 + 0.013\sin(\sqrt{5}k))x_1(k)\\
&+ \frac{(0.025 - 0.001\cos(\sqrt{2}k))x_2(k)}{0.2 - 0.004\cos(\sqrt{3}k) + x_2(k)} + \frac{(0.02 + 0.0015\cos(\sqrt{3}k))x_3(k)}{0.4 + 0.035\cos(\sqrt{2}k) + x_3(k)} - (0.025 - 0.002\cos(\sqrt{3}k))u_1(k)\Big\},\\
x_2(k+1) &= x_2(k)\exp\Big\{1.15 - 0.025\sin(\sqrt{2}k) - (1.085 + 0.015\sin(\sqrt{3}k))x_2(k)\\
&+ \frac{(0.025 + 0.003\cos(\sqrt{5}k))x_1(k)}{0.35 - 0.02\cos(\sqrt{3}k) + x_1(k)} + \frac{(0.025 - 0.002\cos(\sqrt{2}k))x_3(k)}{0.2 + 0.04\sin(\sqrt{3}k) + x_3(k)} - (0.025 + 0.004\sin(\sqrt{2}k))u_2(k)\Big\},\\
x_3(k+1) &= x_3(k)\exp\Big\{1.25 - 0.03\sin(\sqrt{5}k) - (1.1 - 0.024\sin(\sqrt{2}k))x_3(k) \qquad (5.1)\\
&+ \frac{(0.03 - 0.002\cos(\sqrt{2}k))x_1(k)}{0.2 + 0.003\sin(\sqrt{3}k) + x_1(k)} + \frac{(0.028 + 0.0015\cos(\sqrt{3}k))x_2(k)}{0.25 - 0.04\cos(\sqrt{5}k) + x_2(k)} - (0.02 + 0.002\cos(\sqrt{3}k))u_3(k)\Big\},
\end{aligned}$$

$$\begin{aligned}
\Delta u_1(k) &= -(0.93 - 0.03\sin(\sqrt{2}k))u_1(k) + (0.015 + 0.005\sin(\sqrt{3}k))x_1(k)\\
&\quad + (0.013 - 0.004\sin(\sqrt{3}k))x_2(k) + (0.024 - 0.005\cos(\sqrt{5}k))x_3(k),\\
\Delta u_2(k) &= -(0.924 - 0.04\sin(\sqrt{3}k))u_2(k) + (0.018 - 0.004\sin(\sqrt{5}k))x_1(k)\\
&\quad + (0.015 - 0.005\cos(\sqrt{2}k))x_2(k) + (0.014 + 0.004\sin(\sqrt{2}k))x_3(k),\\
\Delta u_3(k) &= -(0.936 - 0.035\cos(\sqrt{5}k))u_3(k) + (0.017 - 0.006\cos(\sqrt{2}k))x_1(k)\\
&\quad + (0.013 - 0.005\sin(\sqrt{3}k))x_2(k) + (0.014 + 0.005\cos(\sqrt{2}k))x_3(k).
\end{aligned}$$

By simple computation, we derive

$$\rho_1 \approx 0.392,\quad \rho_2 \approx 0.542,\quad \rho_3 \approx 0.214,\quad \varphi_1 \approx 0.131,\quad \varphi_2 \approx 0.281,\quad \varphi_2 \approx 0.372.$$

It is easy to see that the conditions of Theorem 4.1 are verified. Therefore, system (5.1) has a unique positive almost periodic solution which is globally attractive. Our numerical simulations support our results (see Fig. 1).

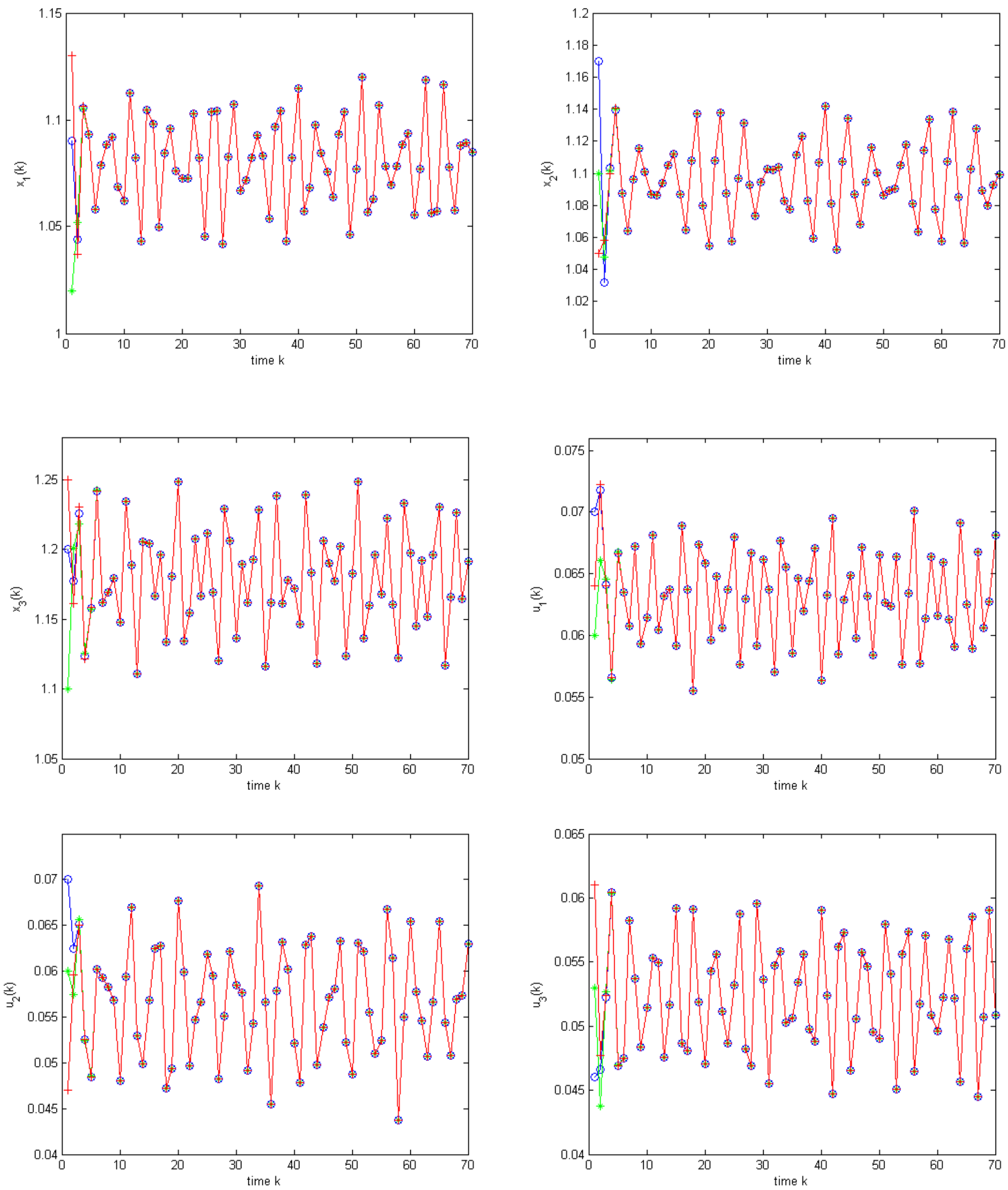

Fig. 1. Dynamic behavior of positive almost periodic solution $(x_1(k), x_2(k), x_3(k), u_1(k), u_2(k), u_3(k))$ of system (5.1) with the three initial conditions(1.02,1.1,1.1,0.06,0.06,0.053), (1.09,1.17,1.2,0.07,0.07,0.046) and (1.13,1.05,1.25,0.064,0.047,0.061) for $k \in [1, 70]$, respectively.

6 Concluding Remarks

In this paper, assuming that the coefficients in system (1.1) are bounded non-negative almost periodic sequences, we obtain the sufficient conditions for the existence of a unique almost periodic solution which is globally attractive. By comparative analysis, we find that when the coefficients in system (1.1) are almost periodic, the existence of a unique almost periodic solution of system (1.1) is determined by the global attractivity of system (1.1), which implies that there is no additional condition to add.

Furthermore, for the almost periodic discrete multispecies Lotka-Volterra mutualism system (1.1) with time delays and feedback controls, we would like to mention here the question of how to study the almost periodicity of the system and whether the existence of a unique almost periodic solution is determined by the global attractivity of the system or not. It is, in fact, a very challenging problem, and we leave it for our future work.

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Competing Interests

The author declares that no competing interests exist.

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