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# DBOLR Test for Testing Autoregressive Conditional Heteroscedasticity and Comparative Study with Two Sided Likelihood Ratio (LR) and Lagrange Multiplier (LM) Tests - A Simulation Approach

Nahida Afroz<sup>1\*</sup> and Hossain Md. Alhelal<sup>2</sup>

<sup>1</sup>Department of statistics, Comilla University, Comilla, Bangladesh.

<sup>2</sup>Department of Monetary Policy, Bangladesh Bank, Bangladesh.

## Authors' contributions

*This work was carried out in cooperation between both authors. Author HMA designed the study, wrote the literature, and first draft of the manuscript. Author NA carried out the analysis and reviewed the draft manuscript. Both authors read and approved the final manuscript.*

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## ABSTRACT

Many econometric testing problems are potentially either one-sided or partially one-sided because econometric models often come with prior information about the sign of some or all of their unknown parameters. In most cases, time series data suffer from both heteroscedasticity and autocorrelation, which is referred to as autoregressive conditional heteroscedasticity (ARCH) effects. This ARCH usually occurs in financial time series data. Usual two-sided LM and LR type tests are not suitable for testing restricted ARCH effect. In this paper we propose one-sided LR tests for testing ARCH effect in the disturbances of a regression model and compare this with the usual two-sided LR and LM tests. Monte Carlo study indicates that the proposed one-sided LR test performs better than the existing two-sided LR, LM tests.

*Keywords: Monte Carlo Simulation; ARCH; LM; LR; Distance-based test.*

\*Corresponding author: E-mail: [nahidafrz@yahoo.com](mailto:nahidafrz@yahoo.com);

## 1. INTRODUCTION

Many econometrics models provide us with prior information about some or all of their unknown parameters. Such information's usually come from economic theory, from previous empirical studies or from functional considerations such as variances always being negative. For this reason, many econometric testing problems are potentially either strictly one-sided or partially one-sided. For example, variance of error components in panel data model with individual and time error components can be expected to be positive. So one-sided or partially one-sided parameter information can be important to help improve the quality of inferences.

One of the important assumptions of classical linear regression model (CLRM) is that errors are normally, independently and identically distributed with mean zero and constant variance. If this assumption is not satisfied, that is if the value of any disturbance term in any particular period is correlated with its own preceding value, then we say that autocorrelation is present in the random variable(s) and if variance of the disturbance is not constant, then we say that there is heteroscedasticity. Now if the current disturbance variance is autocorrelated with previous disturbance terms, then we say that autoregressive conditional heteroscedasticity (ARCH) present in the model.

Breusch and Pagan [1] developed a test based on LM test for a broad class of heteroscedastic specification of disturbances in linear regression model. The test statistic for testing the hypothesis,

$$H_0 : \alpha = 0, \quad (1.1)$$

Against a two-sided alternative

$$H_1 : \alpha \neq 0, \quad (1.2)$$

$$\text{Is } LM = \{s(\theta_0)\}' \{I(\theta_0)\}^{-1} \{s(\theta_0)\}, \quad (1.3)$$

where,  $s(\theta_0)$  and  $I(\theta_0)$  are the score and the information matrix under the null hypothesis, respectively. The asymptotic null hypothesis distribution of (1.3) follows a central chi-square distribution with  $m$  degrees of freedom.

Since two-sided LM test is not appropriate when the sign of the parameters are known. In context

of ARCH model, the conditional variance of the disturbance term is a function of past squared disturbance term up to a specified lag. To ensure that the conditional variance is strictly positive for all realization of disturbances term requires that the parameter space be nonnegative. So in testing the ARCH effect, the alternative hypothesis may be strictly one-sided. So we have to consider the one sided hypothesis and for testing this one-sided hypothesis we maximize the objective function by imposing the positive constraints to the parameter. Since there is no literature on the one-sided LR test, in this paper, our main objective is to develop one-sided version of the LR test (DBOLR) for testing ARCH effect in the disturbance term of a linear regression model by using Distance-based approach (see for example, Majumder [2] and King [3]) and make a comparative study with the conventional counterparts.

In this paper, our main objective is to develop one-sided version of the LR test (DBOLR) for testing ARCH effect in the disturbance term of a linear regression model by using Distance-based approach (see for example, Majumder [2]) and make a comparative study with the conventional counterparts.

## 2. MODEL AND HYPOTHESES

The ARCH model introduced by Engle [4], have been particularly popular and useful in modeling the disturbance behavior of regression models for monetary and financial variables. Let us consider the following regression model,

$$y = X\beta + u \quad (2.1)$$

where,  $y$  is  $(n \times 1)$ ,  $X$  is  $(n \times k)$  and  $\beta$  is a  $(k \times 1)$  vector of parameters. The  $(n \times 1)$  disturbance vector,  $u$  is normally distributed with  $E(u) = 0$  and  $E(u'u) = \sigma_t^2$ . And

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \dots + \alpha_p u_{t-p}^2 \quad (2.2)$$

A variance (2.2) can arise from a disturbance defined as

$$u_t = \varepsilon_t [\alpha_0 + \alpha_1 u_{t-1}^2 + \dots + \alpha_p u_{t-p}^2]^{1/2} \quad (2.3)$$

Where,  $\varepsilon_t \sim N(0,1)$ , then,  $u_t$  is called autoregressive conditional heteroscedasticity of order  $p$  (ARCH ( $p$ )).

In order to test ARCH in the regression model as defined in (2.1) with disturbance term  $u_t$  which follows an ARCH (p) process of the form (2.3). The hypotheses are as follows

$$H_0 : \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \cdot \\ \cdot \\ \alpha_p \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{pmatrix} \text{ vs. } H_1 : \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \cdot \\ \cdot \\ \alpha_p \end{pmatrix} > \begin{pmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{pmatrix} \quad (2.4)$$

### 3. DISTANCE BASED ONE-SIDED LR TEST

In order to test the hypothesis (2.4) we have to minimize ARCH model (2.3)

$$\text{Min}_{\alpha} \sum (e_t - E(e_t))^2$$

subject to the restriction  $\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \cdot \\ \cdot \\ \alpha_p \end{pmatrix} > \begin{pmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{pmatrix}$  where,  $e_t$

is the OLS residuals computed from (2.1)

So, the two-sided LR test is not appropriate when the alternative hypothesis becomes one-sided. In order to construct the two-sided LR test, we estimate the parameter to determine the Likelihood function under the null hypothesis and under the alternative hypothesis and then we take the ratio of them. But these estimates are not optimum. In our DBOLR test we have to estimate the optimum values of  $\theta$  according to the general formulation of distance-based approach, subject to the restrictions,

$$H_a : \alpha > 0, \text{ where, } \alpha \text{ is a } (p \times 1) \text{ matrix}$$

Here we estimate the optimum value of  $\theta$  and then we calculate  $\log(\tilde{\theta})$ . Now the distance-based one-sided LR test rejects the null hypothesis for large values of,

$$-2[L(\tilde{\theta}_R) - L(\tilde{\theta}_{UR})] \sim \chi_p^2 \quad (3.1)$$

where  $\tilde{\theta}$  is the optimized value subject to the restriction  $H_a$ . Under the null hypothesis the distribution of the test statistic (3.1) follows asymptotically weighted mixture of chi-square distribution with  $p$  degrees of freedom. (See for example, Kodde and Palm [5], Shaprio [6], Majumder [2]).

### 4. MONTE CARLO SIMULATION

Monte Carlo simulations are carried out to compare the powers of the two-sided LR test and the newly proposed DBOLR test for detecting ARCH effect in the disturbances of a regression model of the form (2.1). Here, we use real explanatory variables (X). In order to carry out Monte Carlo simulation we generate the following second and third order ARCH disturbance

$$u_t = \varepsilon_t [\alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2]^{1/2}, \quad (4.1)$$

$$u_t = \varepsilon_t [\alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \alpha_3 u_{t-3}^2]^{1/2}, \quad (4.2)$$

where,  $\varepsilon_t \sim N(0,1)$ , and consequently we generate the model (2.1). We perform 10000 replications to calculate (size corrected) simulated powers of the new and existing test.

#### 4.1 Experimental Design

As in this paper we are studying about a newly proposed DBOLR test, for this to investigate the power properties of these tests we use different data sets.

In order to compare the power properties of the DBOLR and LR test for testing ARCH we use four different types of design matrices of real financial and time series data. We compare the powers of the DBOLR test with the usual two-sided LR test. We use the following design matrices:

ARCHD1: A constant dummy and the monthly all share price index covering the period 2003-2008(DSE).

ARCHD2: A constant dummy and annual data for 1960-1995 of USA on the money supply  $M$  (in billions of dollars) and Federal deficit  $D$  (in billions of current dollars) (See Ramanathan [7]).

ARCHD3: A constant dummy and the total GDP of Bangladesh (Covering the period 1975 to 2005).

For testing the maintained hypothesis, all the design matrices ARCHD1, ARCHD2, and ARCHD3 are employed. The following X matrices are used in the experiment.

- X1: A constant dummy and one real variable of ARCHD1,
- X2: A constant dummy and last two real variables of ARCHD2,
- X3: A constant dummy and one real variables of ARCHD3,

We perform our experiment for testing ARCH for different values of the parameters  $\alpha_i$ ,  $i = 1, 2, \dots, p$  (0 to 0.9). For the second-order ARCH scheme (4.1) we estimate simulated powers for testing strictly one-sided hypothesis,

$$H_0^1 : \alpha = 0 \text{ vs. } H_a^1 : \alpha > 0. \text{ (4.1.1)}$$

and we use the above X1, X2, X3 matrices for different values of n. Also we use selected values of  $\alpha_1$  and  $\alpha_2$  0(0.1)0.9 . Considering the

restriction on covariance stationary, i.e.  $\sum_{i=1}^p \alpha_i < 1$

### 5. RESULTS

Here we compare the power of the existing test and newly developed DBOLR tests for testing  $H_a$ , in the context of linear regression model (2.1). The estimated simulated powers of these tests are presented in Tables 1-3.

In order to investigate the power of ARCH test we first consider sample sizes n=30 for the three sets of X-matrices, X1,X2 and X3 for testing strictly one-sided hypothesis  $H_a$  and the simulated results are presented in Table 1. We use one regressor in the first set, which is the all share price index. In the second data set we use two regressors of variable money supply and Federal deficit and finally, in the third data set we use one regressor, which is the total GDP of Bangladesh (Covering the period 1975 to 2005). We observe that the powers of the DBOLR test are much higher than LR test. For example, for n=30,  $\alpha_1 = 0.05$  and  $\alpha_2 = 0.1$  the corresponding values of the LR and DBOLR tests are respectively **0.153** and **0.402** and Fig.1 Fig. 2 and Fig. 3.1 shows the power curve of DBOLR and LR test. From these we can say that DBOLR test is more powerful than LR test.

**Table 1. The power of LR and DBOLR tests of hypothesis  $H_0^1 : \alpha = 0$  vs.  $H_a^1 : \alpha > 0$  under 1<sup>st</sup> order ARCH scheme, using all share price index covering the period (2003--2008) (DSE), annual data for 1960-1995 of USA on the Federal reserve discount rate  $r$  (in percent), money supply  $M$  (in billions of dollars) and Federal deficit  $D$  (in billions of current dollars), the total GDP of Bangladesh (Covering the period 1975 to 2005) for n=30**

		n=30					
		k=1		k=2		k=1	
$\alpha_1$	$\alpha_2$	LR	DBOLR	LR	DBOLR	LR	DBOLR
0	0	0.05	0.05	0.05	0.05	0.05	0.05
0	0.05	0.067	0.144	0.067	0.135	0.065	0.152
0	0.08	0.089	0.222	0.097	0.214	0.091	0.227
0	0.1	0.099	0.27	0.124	0.248	0.111	0.278
0.05	0	0.066	0.134	0.074	0.127	0.073	0.135
0.05	0.05	0.089	0.273	0.092	0.245	0.105	0.271
0.05	0.08	0.125	0.36	0.14	0.317	0.139	0.35
0.05	0.1	0.153	0.402	0.161	0.37	0.164	0.395
0.1	0	0.121	0.269	0.131	0.249	0.13	0.262
0.1	0.05	0.162	0.41	0.176	0.37	0.168	0.405
0.1	0.08	0.207	0.49	0.192	0.442	0.204	0.475
0.1	0.1	0.22	0.54	0.21	0.499	0.224	0.531

**Table 2. The power of LR and DBOLR tests of hypothesis,  $H_0^1 : \alpha = 0$  vs.  $H_a^1 : \alpha > 0$  under 2nd order ARCH scheme, using all share price index covering the period 2003-2008(DSE) for different values of n such as n=30,40,50**

$\alpha_1$	$\alpha_2$	n=30		n=40		n=50	
		LR	DBOLR	LR	DBOLR	LR	DBOLR
0	0	0.05	0.05	0.05	0.05	0.05	0.05
0	0.05	0.067	0.144	0.077	0.174	0.125	0.218
0	0.08	0.089	0.222	0.124	0.285	0.208	0.341
0	0.1	0.099	0.27	0.153	0.346	0.254	0.41
0.1	0.15	0.162	0.39	0.251	0.489	0.361	0.607
0.1	0.18	0.189	0.441	0.291	0.557	0.426	0.676
0.1	0.2	0.205	0.473	0.32	0.596	0.454	0.712
0.1	0.25	0.234	0.554	0.358	0.658	0.513	0.753
0.2	0	0.243	0.477	0.328	0.579	0.453	0.671
0.2	0.05	0.283	0.597	0.394	0.715	0.52	0.817
0.2	0.08	0.298	0.652	0.431	0.772	0.568	0.852
0.2	0.1	0.31	0.681	0.441	0.793	0.598	0.862

**Table 3. The power of LM, LR, DBOLR tests of hypothesis  $H_0^1 : \alpha = 0$  vs.  $H_a^1 : \alpha > 0$  under 1st order ARCH scheme, using all share price index covering the period (2003-2008) (DSE) and annual data for 1960-1995 of USA on the Federal reserve discount rate  $r$  (in percent), money supply  $M$  (in billions of dollars) and Federal deficit  $D$  (in billions of current dollars) for sample sizes n=30**

$\alpha_1$	n=30					
	k=1			k=2		
	LM	LR	DBOLR	LM	LR	DBOLR
0	0.05	0.05	0.05	0.05	0.05	0.05
0.05	0.0615	0.13	0.166	0.0645	0.132	0.144
0.08	0.076	0.178	0.23	0.077	0.16	0.21
0.1	0.084	0.202	0.28	0.0845	0.186	0.264
0.15	0.1055	0.298	0.4	0.104	0.268	0.374
0.18	0.1205	0.322	0.452	0.118	0.29	0.43
0.2	0.134	0.336	0.488	0.13	0.314	0.456
0.25	0.156	0.37	0.556	0.1595	0.344	0.518
0.3	0.1865	0.394	0.592	0.1875	0.358	0.548
0.35	0.219	0.414	0.616	0.2065	0.39	0.578
0.4	0.2425	0.428	0.644	0.23	0.4	0.584
0.45	0.2625	0.426	0.654	0.2515	0.416	0.596
0.5	0.283	0.43	0.65	0.2725	0.416	0.596
0.55	0.304	0.432	0.648	0.295	0.408	0.598

For examining the power performance for large sample size we use n=50 and the above X1, X2, X3 matrices. The estimated simulated results are presented in Table 2. We observe from the tables and Figures that the simulated powers of the DBOLR test are always higher than the LR test in all cases. Especially it is very high near null value(s). For example, for  $\alpha_1 = 0.05$  and  $\alpha_2 = 0.1$ , the value of the LR and DBOLR are

0.226 and 0.505 for n=40 and 0.355 and 0.607 for n=50 respectively.

Performance of one-sided LR as compared to the LM and LR tests for testing  $H_a$  (ARCH) in the context of linear regression model (2.1). The estimated simulated powers of these tests are presented in Table-3. For example, for  $\alpha_1 = 0.05$  and n=30 value of LM, LR and DBOLR test are

**0.0615, 0.13 and 0.166** respectively. From Table 4 it is also observed that for  $n=30$ ,  $\alpha_1 = 0.05$  and  $\alpha_2 = 0.1$  the simulated power of the LM, LR and DBOLR are **0.069, 0.153 and 0.402** respectively. Again for  $n=40$ ,  $\alpha_1 = 0.05$

and  $\alpha_2 = 0.1$  the simulated power of the LM, LR and DBOLR are **0.073, 0.226 and 0.505** respectively. From Fig. 3.2 and Fig.4 also shows the same result that is power curve of DBOLR test is higher than LM and LR test.

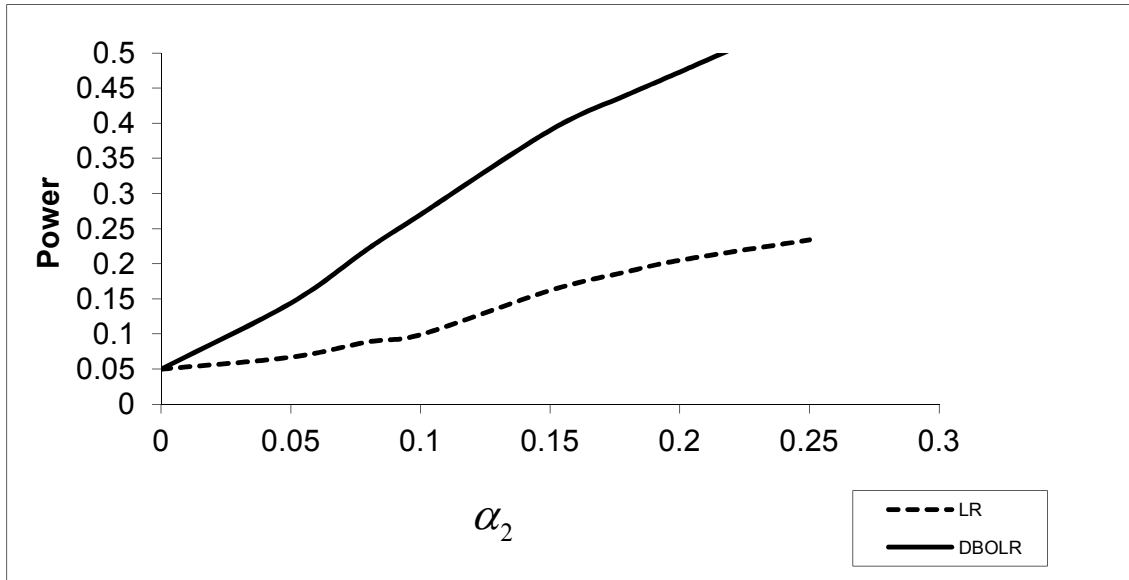


Fig. 1. Power curve of LR and DBOLR tests for hypothesis H1, for  $n=30$ ,  $k=1$ ,  $p=2$  and for fixed  $\alpha_1 = 0$

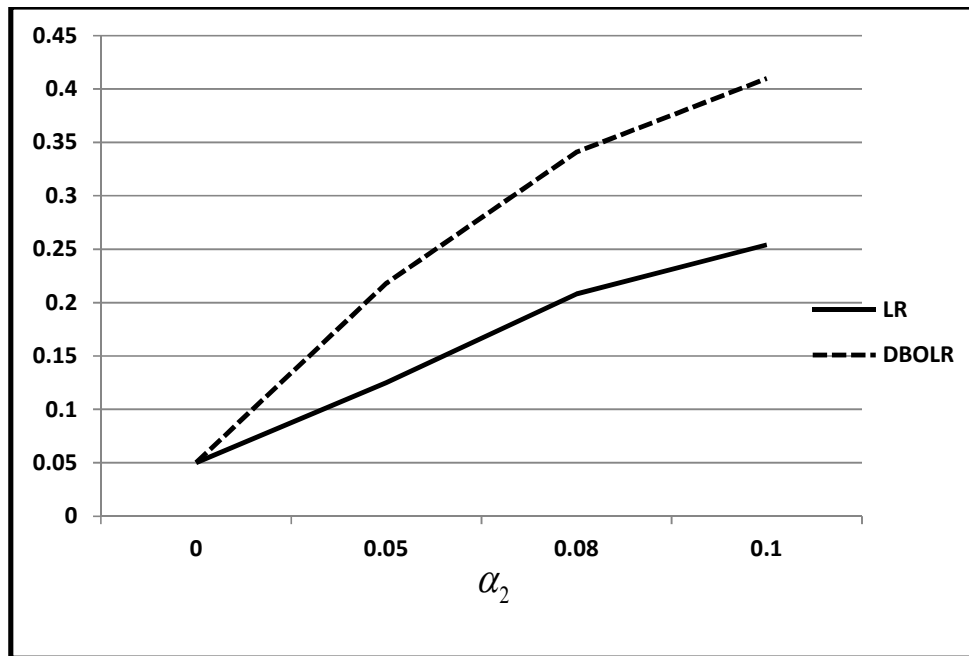


Fig. 2. Power curve of LR and DBOLR tests for hypothesis H1, for  $n=50$ ,  $k=1$ ,  $p=2$  and for fixed  $\alpha_1 = 0$

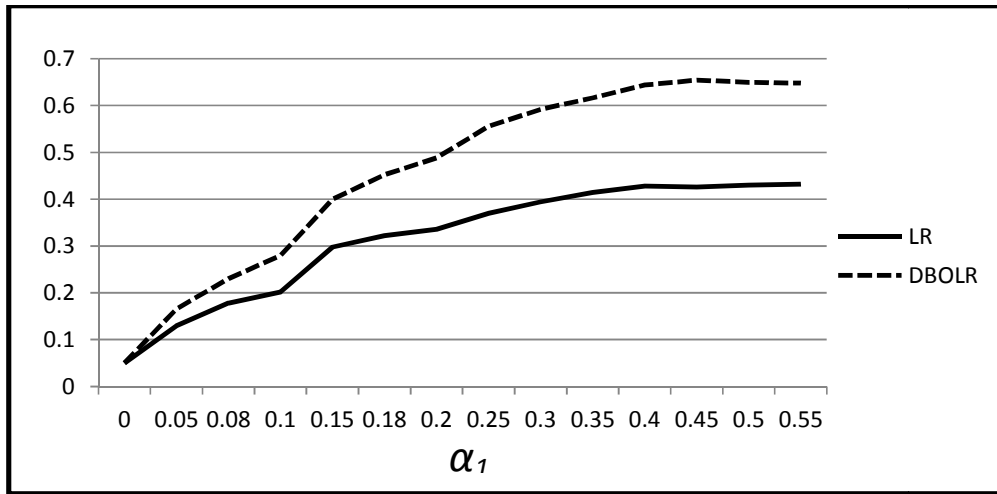


Fig. 3.1. Power curve of LR and DBOLR tests for hypothesis H1, for n=30, k=1, p=1

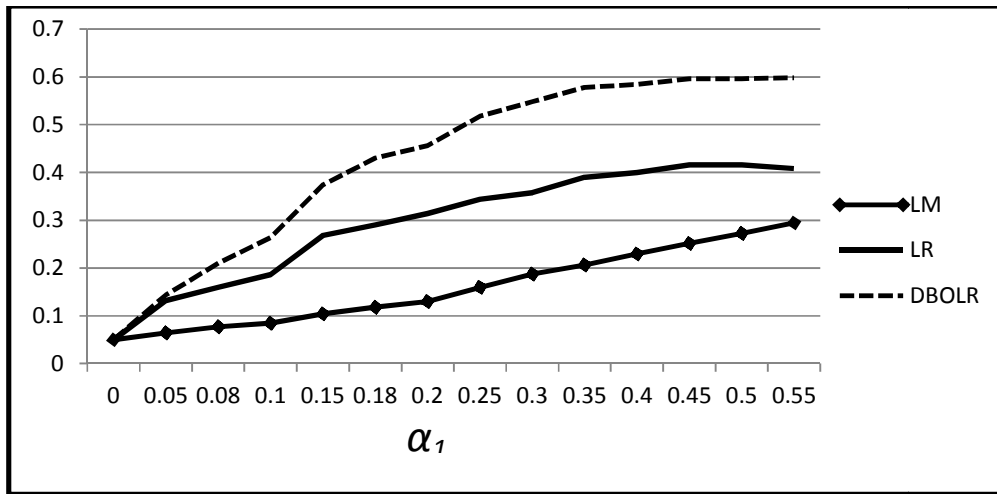


Fig. 3.2. Power curve of LM, LR and DBOLR tests for hypothesis H1, for n=30, k=2, p=1

Table 4. The power of LM, LR and DBOLR tests of hypothesis  $H_0^1: \alpha = 0$  vs.  $H_a^1: \alpha > 0$  under 2<sup>nd</sup> order ARCH scheme, using all share price index covering the period (2003-2008) (DSE), for n=30 and 40

$\alpha_1$	$\alpha_2$	n=30			n=40		
		LM	LR	DBOLR	LM	LR	DBOLR
0	0	0.05	0.05	0.05	0.05	0.05	0.05
	0.05	0.053	0.067	0.144	0.062	0.077	0.174
	0.08	0.056	0.089	0.222	0.067	0.124	0.285
	0.1	0.065	0.099	0.27	0.076	0.153	0.346
	0.15	0.082	0.162	0.39	0.104	0.251	0.489
	0.18	0.091	0.189	0.441	0.122	0.291	0.557
	0.2	0.1	0.205	0.473	0.131	0.32	0.596
0.05	0.25	0.117	0.234	0.554	0.153	0.358	0.658
	0	0.055	0.066	0.134	0.051	0.088	0.172
	0.05	0.062	0.089	0.273	0.067	0.143	0.324

$\alpha_1$	$\alpha_2$	n=30			n=40		
		LM	LR	DBOLR	LM	LR	DBOLR
0.08	0.08	0.063	0.125	0.36	0.072	0.192	0.438
	0.1	0.069	0.153	0.402	0.073	0.226	0.505
	0.15	0.084	0.199	0.527	0.105	0.31	0.645
	0.18	0.09	0.227	0.581	0.122	0.345	0.696
	0.2	0.093	0.237	0.615	0.131	0.37	0.722
	0.25	0.118	0.26	0.664	0.161	0.419	0.772
	0	0.065	0.089	0.217	0.069	0.141	0.263
	0.05	0.068	0.132	0.353	0.077	0.205	0.421
	0.08	0.074	0.185	0.437	0.084	0.245	0.534
	0.1	0.077	0.209	0.49	0.087	0.279	0.595
0.1	0.15	0.091	0.228	0.602	0.116	0.344	0.712
	0.18	0.096	0.258	0.641	0.134	0.382	0.754
	0.2	0.099	0.267	0.658	0.141	0.397	0.776
	0.25	0.117	0.297	0.699	0.168	0.43	0.812
	0	0.072	0.121	0.269	0.083	0.172	0.325
	0.05	0.078	0.162	0.41	0.087	0.244	0.493
	0.08	0.083	0.207	0.49	0.093	0.282	0.596
	0.1	0.085	0.22	0.54	0.1	0.307	0.655
	0.15	0.091	0.255	0.633	0.123	0.37	0.75
	0.18	0.1	0.272	0.667	0.143	0.402	0.784
0.15	0.2	0.111	0.284	0.687	0.157	0.418	0.802
	0.25	0.125	0.308	0.721	0.179	0.454	0.828
	0	0.093	0.191	0.381	0.11	0.249	0.468
	0.05	0.097	0.237	0.529	0.117	0.315	0.626
	0.08	0.096	0.256	0.598	0.126	0.36	0.705
	0.1	0.099	0.268	0.629	0.138	0.383	0.745
	0.15	0.106	0.303	0.685	0.157	0.444	0.797
	0.18	0.118	0.311	0.702	0.173	0.463	0.821
	0.2	0.125	0.316	0.723	0.19	0.471	0.828
	0.25	0.145	0.34	0.754	0.214	0.482	0.847
0.18	0	0.105	0.225	0.449	0.128	0.304	0.536
	0.05	0.112	0.265	0.569	0.141	0.362	0.683
	0.08	0.107	0.282	0.63	0.148	0.404	0.748
	0.1	0.108	0.292	0.655	0.155	0.429	0.786
	0.15	0.117	0.322	0.71	0.178	0.465	0.819
	0.18	0.126	0.337	0.734	0.195	0.478	0.838
	0.2	0.135	0.341	0.742	0.202	0.478	0.849
	0.25	0.153	0.369	0.769	0.223	0.5	0.866
	0	0.113	0.243	0.477	0.137	0.328	0.579
	0.05	0.12	0.283	0.597	0.151	0.394	0.715
0.2	0.08	0.121	0.298	0.652	0.162	0.431	0.772
	0.1	0.118	0.31	0.681	0.169	0.441	0.793
	0.15	0.125	0.342	0.722	0.193	0.48	0.836
	0.18	0.132	0.349	0.747	0.206	0.478	0.844
	0.2	0.139	0.348	0.756	0.207	0.485	0.858
	0.25	0.159	0.375	0.782	0.233	0.513	0.87
	0	0.129	0.28	0.55	0.167	0.38	0.647
	0.05	0.14	0.313	0.661	0.177	0.446	0.764
	0.08	0.14	0.327	0.708	0.187	0.465	0.802
	0.1	0.139	0.336	0.726	0.197	0.472	0.819
0.25	0.15	0.153	0.357	0.751	0.216	0.486	0.845
	0.18	0.161	0.364	0.763	0.229	0.502	0.857
	0.2	0.17	0.379	0.779	0.237	0.521	0.867
	0.25	0.185	0.404	0.797	0.265	0.543	0.877



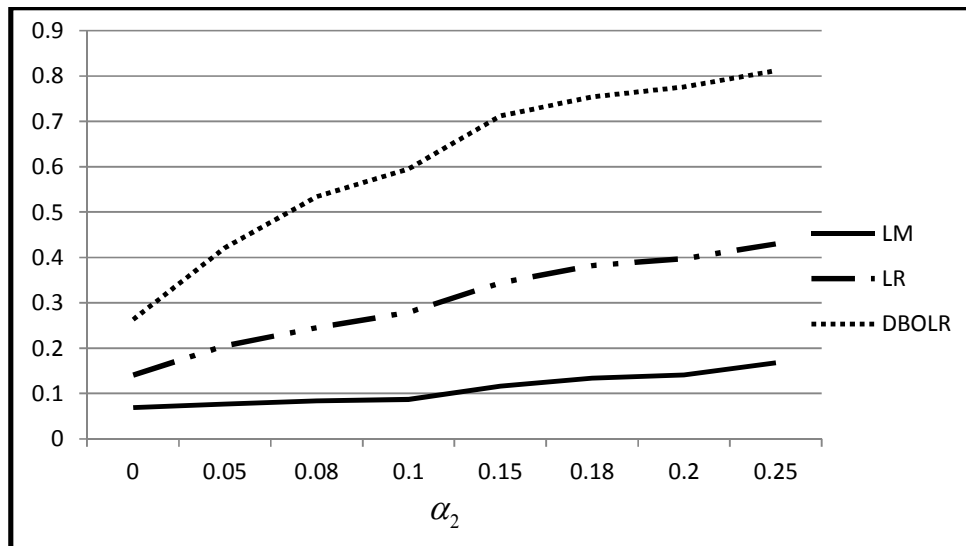


Fig. 4. Power curve of LM, LR and DBOLR tests for hypothesis H1, for  $n=40$ ,  $k=2$ ,  $p=2$  and for fixed  $\alpha_1 = 0.081$

## 6. CONCLUSION

In conclusion we can say that, this paper develops one-sided version of LR type tests for testing higher order ARCH effect in a linear regression model for the strictly one-sided alternatives. Monte Carlo results indicate that newly proposed one-sided version of LR (DBOLR) tests perform better than the usual two sided test and one-sided LM test. In order to compare the performances of Distance Based one-sided LR test with one-sided LM test, we present simulated powers of this test in Table-3. We observe from the table that powers of the DBOLR test are always higher than LR and LM test.

## COMPETING INTERESTS

Authors have declared that no competing interests exist.

## REFERENCES

1. Breusch TS, Pagan AR. a simple test for heteroscedasticity and random coefficient variation. *Econometrica*, Econometric Society. 1979;47(5):1287-94.
2. Majumder AK. One-sided and partially one-sided hypothesis testing in econometrics, an unpublished thesis, Monash University, Australia; 1999.
3. King M. Australian Economic paper. 1981; 163.
4. Engle Robert F. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*. 1982;50:987-100.
5. Kodde DA, Palm FC. Wald criteria for jointly testing equality and inequality restrictions. *Econometrica*. 1986;54:1243-1248.
6. Shapiro A. Towards a unified theory of inequality constrained testing in multivariate analysis. *International Statistical Review*. 1988;56:49-62.
7. Ramanathan R. Introductory Econometrics with applications, 3<sup>rd</sup> ed., Harcourt College Publishers, New York; 1995.

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