



A Novel 5x5 Edge Detection Operator for Blood Vessel Images

Toufic El-Arwadi¹ and Ali El-Zaart^{1*}

¹Department of Mathematics and Computer Science, Faculty of Science, Beirut Arab University, Lebanon.

Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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ABSTRACT

Background: Blood vessel appearance is an important indicator for many diagnoses, including diabetes, hypertension, and arteriosclerosis.

Aims: Blood Vessel edge detection in retinal images is very important in medical image processing. A lot of algorithms have been suggested for extracting medical image edges.

Methodology: In this paper a new 5x5 edge detection masks are proposed based on the finite difference method.

Results: The proposed method is applied on a set of blood vessel images and we obtained good results in comparison with 5x5 Sobel mask.

Keywords: Blood vessel detection; edge detection; 5x5 mask operator; Taylor's expansion.

1. INTRODUCTION

Retinal images of humans play an important role in the detection and diagnosis of many eye

diseases for ophthalmologists [1]. Some diseases such as glaucoma [2,3], diabetic retinopathy, and macular degeneration are very serious for they can lead to blindness if they are

*Corresponding author: E-mail: dr_elzaart@yahoo.com, elzaart@bau.edu.lb;

not detected in time and correctly [4]. Therefore, automatic detection for retinal images is necessary, and among them the detection of blood vessels is most important [6-8]. The information about blood vessels, such as length, width, tortuosity and branching pattern, can not only provide information on pathological changes but can also help to grade diseases severity or automatically diagnose the diseases [5]. The retinal blood vessel detection is one of the preprocessing steps for the detection and diagnosis of these abnormal lesions. Edge detection is an important field in image processing. An edge is detected as the discontinuity in intensity level of a point. The edge information of an image could be used to identify objects in the image, or on the other hand, enhancing the image by sharpening it. Edges are mostly detected using either the first derivatives, called gradient, or the second derivatives, called Laplacian. Laplacian is more sensitive because it is based on the second derivative. The objective of this work is to detect edges in retinal blood vessel image; this is a pre-processing step for blood vessel detection. The blood vessel detection step takes the edge information and tries to reconstruct the entire blood vessel. In this paper we developed new 5x5 masks based on the 1D and 2D Taylor's expansion. The rest of this paper is organized as follows. Section 2 presents the related work of edge detection. Our proposed 5x5 masks for edge detection is described in section 3. Experimental results is shown in section 4. Finally this paper presents conclusion and future work in section 5.

2. RELATED WORKS

An edge is usually a step change in intensity in an image. It corresponds to the boundary between two regions or a set of points in the image where luminous intensity changes very sharply [9]. The decision of whether pixel is an edge point or not bases on how much its local neighbors respond to a certain edge detector [10]. Over the years, many methods have been proposed for detecting edges in images. Some of the earlier methods, such as the Sobel and Prewitt detectors used local gradient operators [10] to obtain spatial filter masks. The procedure is to compute the sum of products of the mask coefficients with the intensity values in the region encompassed by the mask [10]. Also the Canny edge detector which depends on the Gaussian distribution for obtaining the operators for the gradient and Laplacian masks is a well-known edge detector [10]. In this paper we propose 5x5 masks for edge detection in retinal images. The method is based on the 1D and 2D Taylor's expansion.

3. PROPOSED METHOD

The proposed method is based on 2D Taylor expansion. First in section 3.1, we will explain the 1D Taylor's formula and then in section 3.2 we will explain the 2D Taylor's expansion.

3.1 1D Taylor's Formula

In pure mathematics, given a function $f \in C^{n+1}$ that is, all its derivatives $f^{(j)}, j = 1, \dots, n + 1$ are continuous. Let x be a fixed point, then for h close to 0, there exists $t \in (0,1)$ such that:

$$f(x + h) = f(x) + \sum_{i=1}^n \frac{h^i}{i!} f^{(i)}(x) + f^{(n+1)}(c) \frac{h^{n+1}}{(n+1)!} \quad (1)$$

Where $c = x + th$.

The term $R_n := f^{(n+1)}(c) \frac{h^{n+1}}{(n+1)!}$ is called the remainder term. Using the big - 0 notation, we abbreviate the formula (1) as follow:

$$f(x + h) = f(x) + \sum_{i=1}^n \frac{h^i}{i!} f^{(i)}(x) + O(h^{n+1}) \quad (2)$$

3.2 2D Taylor's Formula

Let X be a normed vector space, If $f: X \rightarrow \mathbb{R}$ is differentiable $n + 1$ times on X , it may be expanded by Taylor's formula, for each $x \in X$:

$$f(x + h) = f(x) + Df(x).h + \frac{1}{2!}D^2f(x).h^2 + \dots + \frac{1}{n!}D^n f(x).h^n + R_n(x) \quad (3)$$

Where $R_n(x) = \frac{1}{(n+1)!}D^{n+1}f(x).h^{n+1}$.

The k-Frechet Derivative of f at x is being denoted by $D^k f(x)$, to be viewed as a multilinear map from X^k to R. The $.h^k$ notation means to evaluate $D^k f(x)$ at (h, \dots, h) .

If $X = R^2$, D^k has the following expression $\frac{1}{k!}D^k f(x).h^k = \sum_{|J|=k} \frac{1}{J!} \frac{\partial^{|J|} f}{\partial x^J}(x)h^J$

Where J is a vector of 2 components. The multi-index J runs through all combinations such that

$J_1 + J_2 = |J| = k$ in the sum and $J! = J_1! J_2!$

From equation (3), we can rewrite the 2D Taylor expansion as follows:

$$f(x + l, y + k) = f(x, y) + l \frac{\partial f}{\partial x} + k \frac{\partial f}{\partial y} + \frac{1}{2} \left(\frac{\partial^2 f}{\partial x^2} l^2 + 2hl \frac{\partial^2 f}{\partial x \partial y} + \frac{\partial^2 f}{\partial y^2} k^2 \right) + O(l^3 + k^3) \quad (4)$$

From this formula, we take the values of $l = (-1)^n h$ and $k = (-1)^m h$, for $n = \pm 1$ and $m = \pm 1$ to obtain the following 4 formulas :

-For $l = h$ and $k = h$, we obtain:

$$f(x + h, y + h) = f(x, y) + h \frac{\partial f}{\partial x} + h \frac{\partial f}{\partial y} + \frac{h^2}{2} \left(\frac{\partial^2 f}{\partial x^2} + 2 \frac{\partial^2 f}{\partial x \partial y} + \frac{\partial^2 f}{\partial y^2} \right) + O(h^3) \quad (5)$$

- For $l = -h$ and $k = h$, we obtain:

$$f(x - h, y + h) = f(x, y) - h \frac{\partial f}{\partial x} + h \frac{\partial f}{\partial y} + \frac{h^2}{2} \left(\frac{\partial^2 f}{\partial x^2} - 2 \frac{\partial^2 f}{\partial x \partial y} + \frac{\partial^2 f}{\partial y^2} \right) + O(h^3) \quad (6)$$

- For $l = h$ and $k = -h$, we obtain:

$$f(x + h, y - h) = f(x, y) + h \frac{\partial f}{\partial x} - h \frac{\partial f}{\partial y} + \frac{h^2}{2} \left(\frac{\partial^2 f}{\partial x^2} - 2 \frac{\partial^2 f}{\partial x \partial y} + \frac{\partial^2 f}{\partial y^2} \right) + O(h^3) \quad (7)$$

- For $l = -h$ and $k = -h$, we obtain:

$$f(x - h, y - h) = f(x, y) - h \frac{\partial f}{\partial x} - h \frac{\partial f}{\partial y} + \frac{h^2}{2} \left(\frac{\partial^2 f}{\partial x^2} + 2 \frac{\partial^2 f}{\partial x \partial y} + \frac{\partial^2 f}{\partial y^2} \right) + O(h^3) \quad (8)$$

By multiplying the equations (5) and (7) by α and the equation (6) and (8) by β , we obtain the following formula:

$$\begin{aligned} &\alpha f(x + h, y + h) + \alpha f(x + h, y - h) + \beta f(x - h, y + h) + \beta f(x - h, y - h) \\ &= (2\alpha + 2\beta)f(x, y) + (2\alpha - 2\beta)h \frac{\partial f}{\partial x} + (\alpha + \beta)h^2 \Delta f + O(h^3) \end{aligned} \quad (9)$$

Where Δf is the Laplacian of $f(x, y)$. By taking h close to zero, we can neglect h^2 and h^3 and we deduce the following approximation for $\frac{\partial f}{\partial x}$

$$\frac{\partial f}{\partial x} = \frac{1}{h(2\alpha - 2\beta)} (\alpha f(x+h, y+h) + \alpha f(x+h, y-h) + \beta f(x-h, y+h) + \beta f(x-h, y-h) - (2\alpha + 2\beta)f(x, y)) \quad (10)$$

By multiplying the equations (5) and (6) by α and the equation (7) and (8) by β , we can obtain the following formula:

$$\alpha f(x+h, y+h) + \alpha f(x-h, y+h) + \beta f(x+h, y-h) + \beta f(x-h, y-h) = (2\alpha + 2\beta)f(x, y) + (2\alpha - 2\beta)h \frac{\partial f}{\partial y} + (\alpha + \beta)h^2 \Delta f + O(h^3) \quad (11)$$

Similarly, we can deduce the following approximation for $\frac{\partial f}{\partial y}$

$$\frac{\partial f}{\partial y} = \frac{1}{h(2\alpha - 2\beta)} (\alpha f(x+h, y+h) + \alpha f(x-h, y+h) + \beta f(x+h, y-h) + \beta f(x-h, y-h) - (2\alpha + 2\beta)f(x, y)) \quad (12)$$

To obtain a 5x5 horizontal mask, we replace h by $2h$ in (10), we obtain the following formula

$$\frac{\partial f}{\partial x} = \frac{1}{h(4\alpha - 4\beta)} (\alpha f(x+2h, y+2h) + \alpha f(x+2h, y-2h) + \beta f(x-2h, y+2h) + \beta f(x-2h, y-2h) - (2\alpha + 2\beta)f(x, y)) \quad (13)$$

Similarly, to obtain a 5x5 vertical mask, replace h by $2h$ in (11), we obtain the following formula

$$\frac{\partial f}{\partial y} = \frac{1}{h(4\alpha - 4\beta)} (\alpha f(x+2h, y+2h) + \alpha f(x-2h, y+2h) + \beta f(x+2h, y-2h) + \beta f(x-2h, y-2h) - (2\alpha + 2\beta)f(x, y)) \quad (14)$$

Now adding equations (10) and (13), we obtain the following formula:

$$\frac{\partial f}{\partial x} = \frac{1}{h(6\alpha - 6\beta)} (\alpha f(x+h, y+h) + \alpha f(x+2h, y+2h) + \alpha f(x+h, y-h) + \alpha f(x+2h, y-2h) + \beta f(x-h, y+h) + \beta f(x-2h, y+2h) + \beta f(x-h, y-h) + \beta f(x-2h, y-2h) - 4\alpha + 4\beta f(x, y)) \quad (15)$$

Similarly, adding equations (12) and (14) leads to the following formula:

$$\frac{\partial f}{\partial y} = \frac{1}{h(6\alpha - 6\beta)} (\alpha f(x+2h, y+2h) + \alpha f(x+h, y+h) + \alpha f(x-h, y+h) + \alpha f(x-2h, y+2h) + \beta f(x+h, y-h) + \beta f(x+2h, y-2h) + \beta f(x-h, y-h) + \beta f(x-2h, y-2h) - (4\alpha + 4\beta)f(x, y)) \quad (16)$$

The function $f(x, y)$ presented in the above formulas is continuous, however in digital image processing $f(x, y)$, which represents an image and should be discrete. For that reason we set the value of h to 1.

$$\frac{\partial f}{\partial x} = \frac{1}{(6\alpha - 6\beta)} (\alpha f(x+1, y+1) + \alpha f(x+2, y+2) + \alpha f(x+1, y-1) + \alpha f(x+2, y-2) + \beta f(x-1, y+1) + \beta f(x-2, y+2) + \beta f(x-1, y-1) + \beta f(x-2, y-2) - 4\alpha + 4\beta f(x, y)) \quad (17)$$

From the above formula (17), we construct the 5x5 horizontal masks as follows:

$$M_x = \frac{1}{(6\alpha - 6\beta)} \begin{pmatrix} \beta & 0 & 0 & 0 & \beta \\ 0 & \beta & 0 & \beta & 0 \\ 0 & 0 & 4(\beta + \alpha) & 0 & 0 \\ 0 & \alpha & 0 & \alpha & 0 \\ \alpha & 0 & 0 & 0 & \alpha \end{pmatrix}$$

Similarly, we can write the My Mask as following:

$$M_y = \frac{1}{(6\alpha - 6\beta)} \begin{pmatrix} \alpha & 0 & 0 & 0 & \beta \\ 0 & \alpha & 0 & \beta & 0 \\ 0 & 0 & -4(\beta + \alpha) & 0 & 0 \\ 0 & \alpha & 0 & \beta & 0 \\ \alpha & 0 & 0 & 0 & \beta \end{pmatrix}$$

The Sobel 5x5 horizontal and vertical mask are [10]

$$M_x = \begin{pmatrix} -1 & -4 & -6 & -4 & -1 \\ -2 & -8 & -12 & -8 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 8 & 12 & 8 & 2 \\ 1 & 4 & 6 & 4 & 1 \end{pmatrix} \text{ and } M_y = \begin{pmatrix} 1 & 2 & 0 & -2 & -1 \\ 4 & 8 & 0 & -8 & -4 \\ 6 & 12 & 0 & -12 & -6 \\ 4 & 8 & 0 & -8 & -4 \\ 1 & 2 & 0 & -2 & -1 \end{pmatrix}$$

4. EXPERIMENTAL RESULTS

In our experimental results section, we applied our proposed 5x5 masks on several blood vessel retina images from the STARE database and in this section we only show three of them. We choose the value of $\alpha=1$ and $\beta=2$ based on the experimentation. Based on these values, the proposed 5x5 masks will be as follows:

$$M_x = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -12 & 0 & 2 \\ 0 & 2 & 0 & 2 & 0 \\ 2 & 0 & 0 & 0 & 2 \end{pmatrix} \text{ and } M_y = \begin{pmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & -12 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 & 2 \end{pmatrix}$$

In Fig. 1, we presented in Fig. 1a a color retinal image 01-test.tif from the STARE database. Then we applied our 5x5 masks in x-direction and in y-direction, the results are presented in Fig. 1b and 1d respectively. In order to detect the edges, we applied threshold value $T=50$ on the images presented in Fig. 1b and 1d. Fig. 1c and 1e presented the edge detection maps of 5x5 x-direction and y-direction. In comparison with the 5x5 Sobel masks, Figs. 1f and 1h presented the results of 5x5 masks in x-direction and y-direction. We applied on these resulted images a threshold value $T=170$ in order to detect the edge in these images. Fig. 1g and 1i presented the edge detection maps of 5x5 Sobel in x-direction and y-direction. In comparison between our result in Fig. 1b and the result of Sobel in Fig. 1f, we remark that sobel operator is more sensitive to noise and the detected edges in sobel results are more thick in comparison with our results. The

same comments for y-direction results. To clarify this point, we put an arrow between our results in Fig. 1e and the sobel results in Fig. 1i. to show the missed edge in Sobel results. From the Fig. 1, we conclude that our proposed method gave better results than the Sobel masks. The same is for Fig. 2 (02-test.tif) and Fig. 3 (04-test.tif). Note that we did not use a performance measure method in order it compare the results, because it is clear that the proposed operators gave better results than the Sobel.

Note that the detected edge in retinal blood vessel image is a pre-processing step for blood vessel detection. In order to show that our results are better than Sobel results, we compare them with the reference images. Fig. 4, shows a comparison between our results and Sobel Results with the reference images.

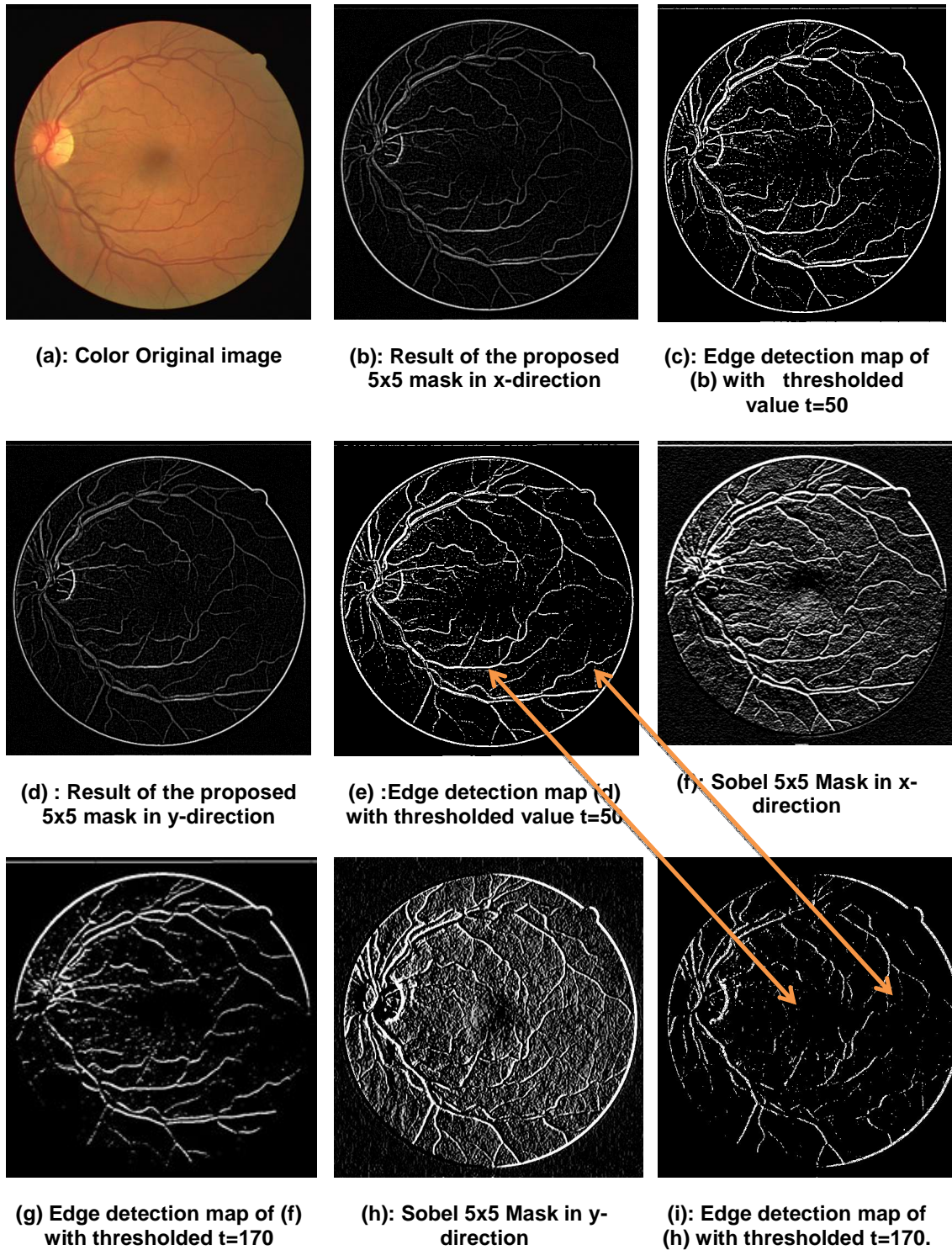


Fig. 1. Results of our proposed method and 5x5 sobel masks

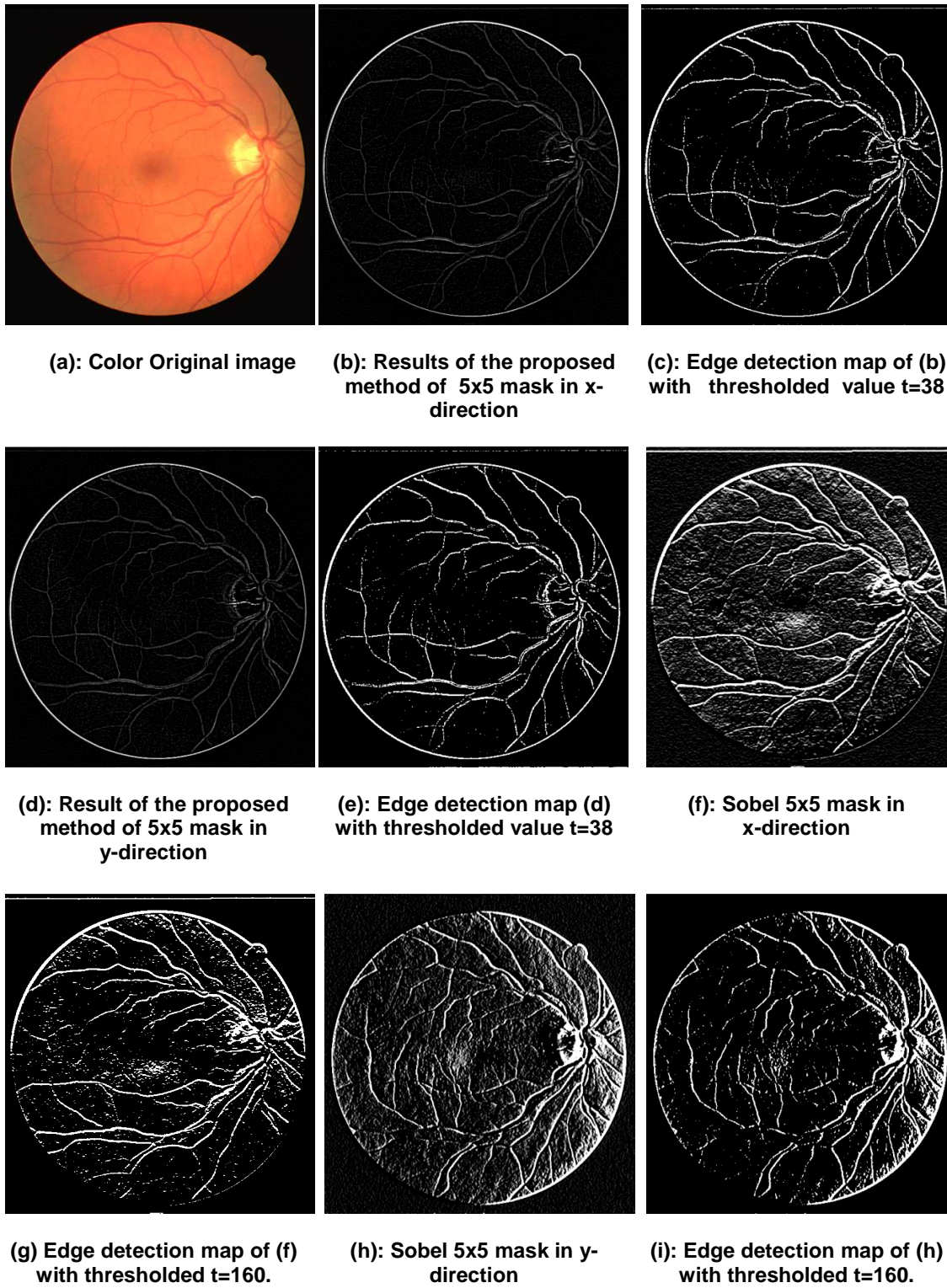
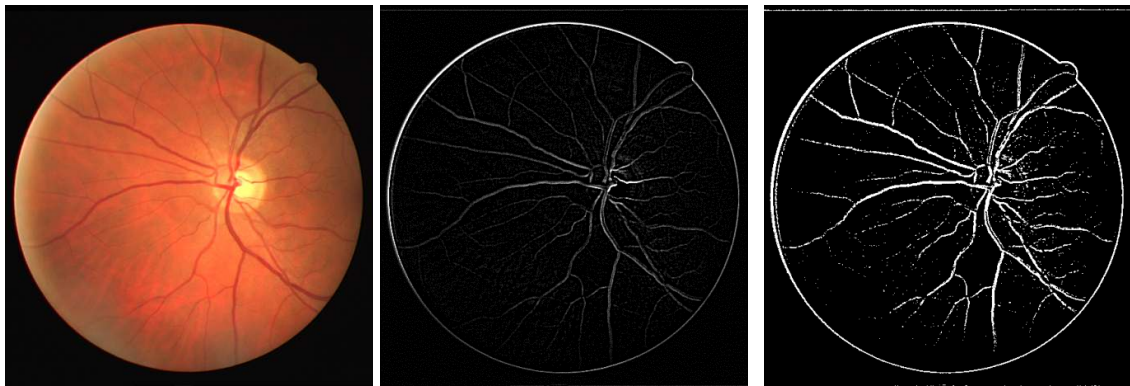


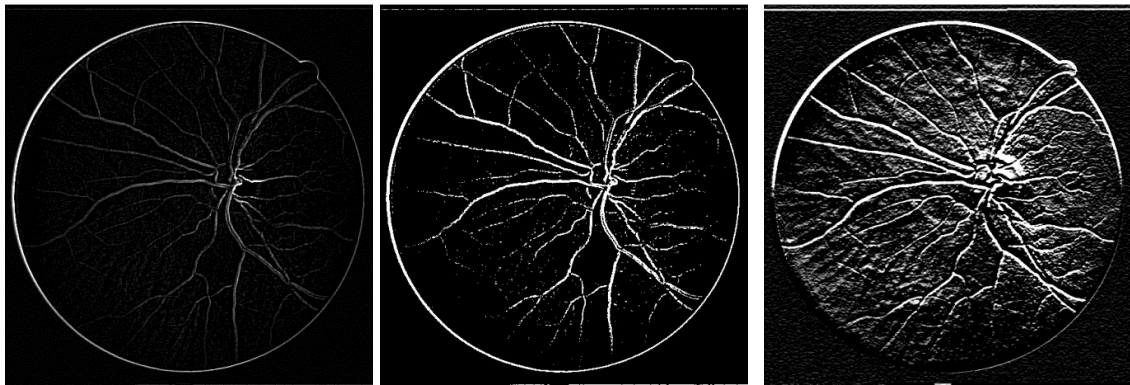
Fig. 2. Results of our proposed method and 5x5 sobel masks



(a): Color Original image

(b): Results of the proposed method of 5x5 mask in x-direction

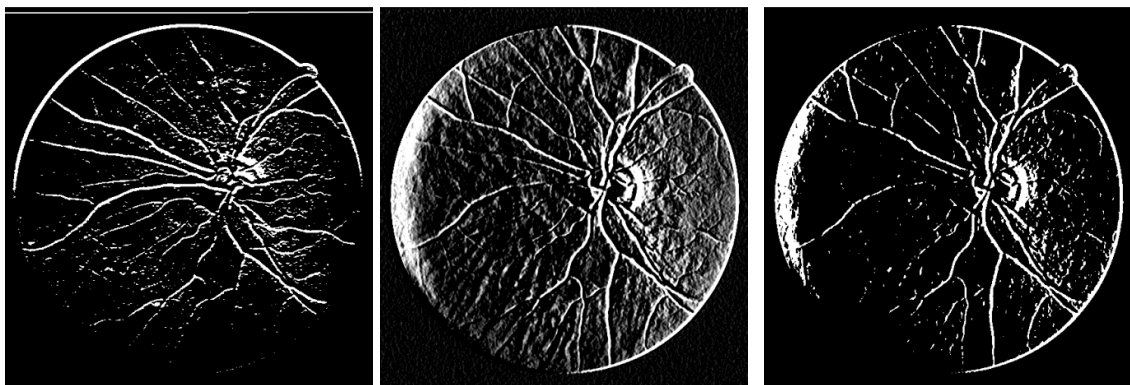
(c): Edge detection map of (b) with thresholded value $t=35$



(d): Result of the proposed method of 5x5 mask in y-direction

(e): Edge detection map (d) with thresholded value $t=35$

(f): Sobel 5x5 mask in x-direction

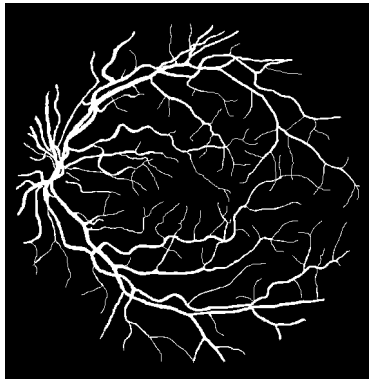


(g): Edge detection map of (f) with thresholded $t=190$.

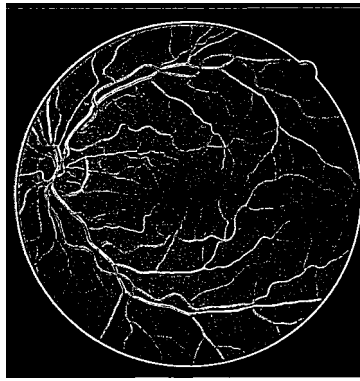
(h): Sobel 5x5 mask in y-direction

(i): Edge detection map of (g) with thresholded $t=190$.

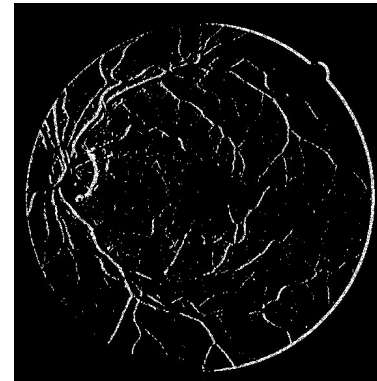
Fig. 3. Results of our proposed method and 5x5 sobel masks



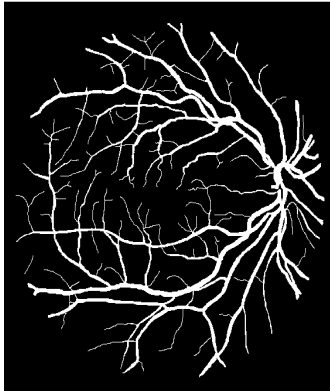
(a): Reference image of 01-test.tif



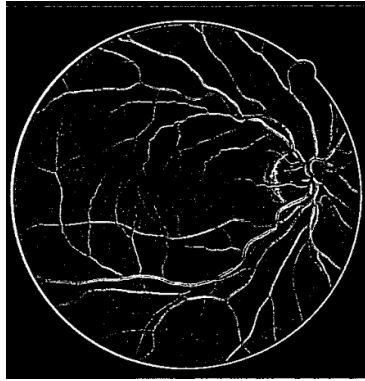
(b): Our results



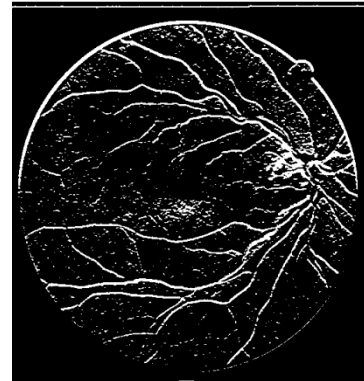
(c): Sobel result



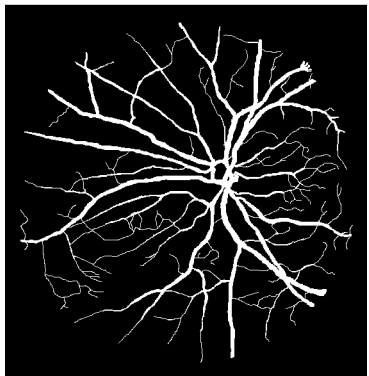
(d): Reference image of 02-test.tif



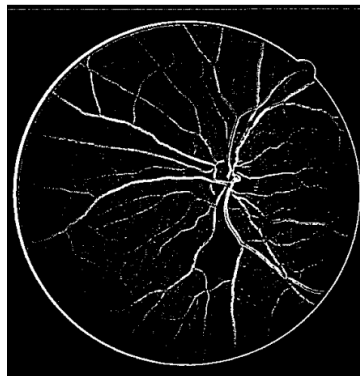
(e): Our result



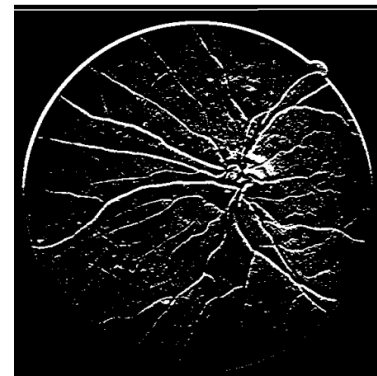
(f): Sobel Result



(g): Reference image of 04-test.tif



(h): Our result



(i): Sobel Result

Fig. 4. Comparison between our results and Sobel results with the reference images

5. CONCLUSION AND FUTURE WORK

In this paper, we presented a new method for detecting edges in blood vessel retinal images. We constructed 5x5 masks from the Taylor's expansion. The new masks are applied on blood vessel retinal images and we obtained good results in comparison with Sobel 5x5 masks.

The selection of the values of α and β was done manually, that will increase the time complexity. As future work, an optimization method is needed in order to select the best choice of the value of α and β . The objective of the proposed method is to detect edges in retinal blood vessel image, as future work, we have to develop another method in order to reconstruct the entire blood vessel from detected edges.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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